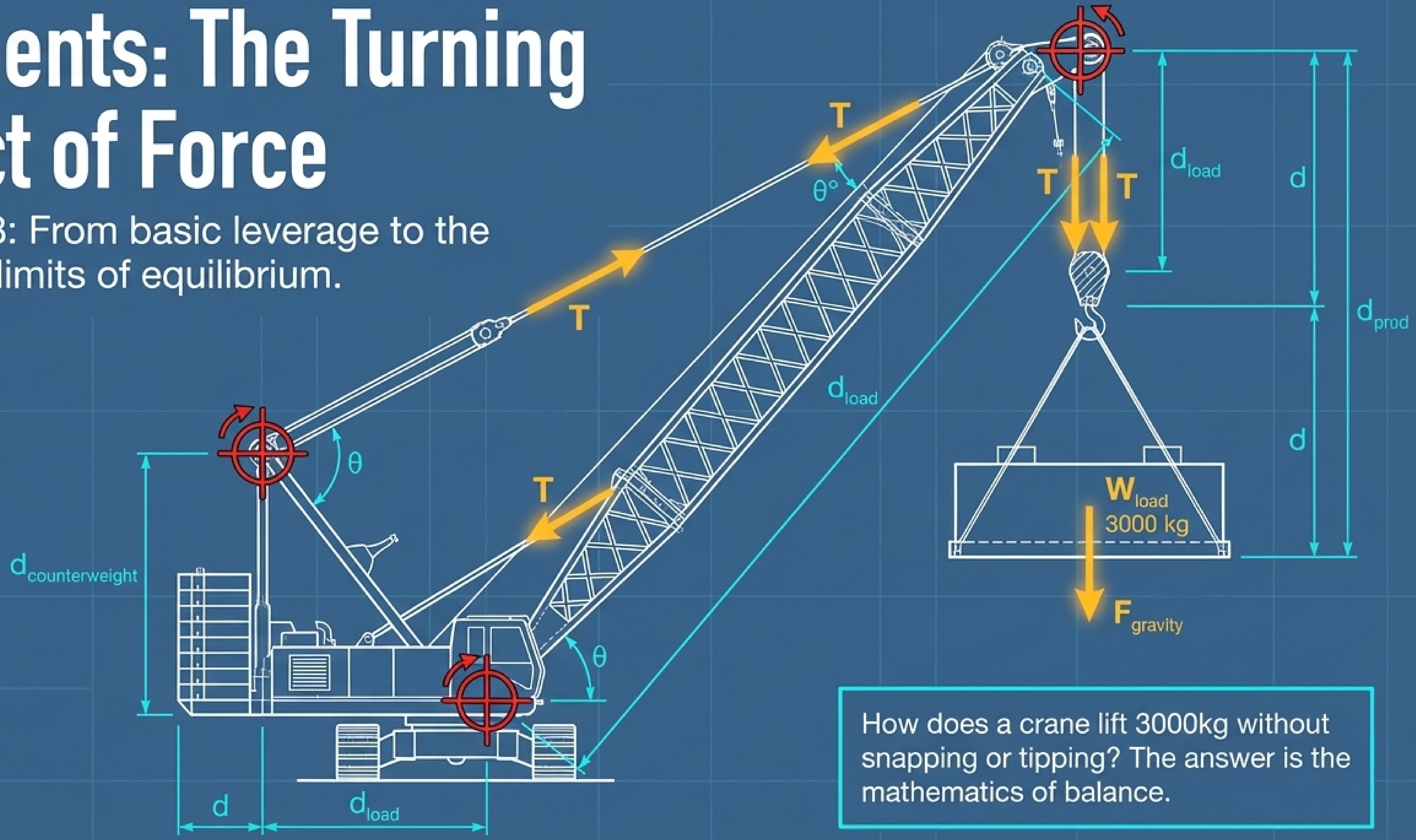
Moments: The Turning Effect of Force

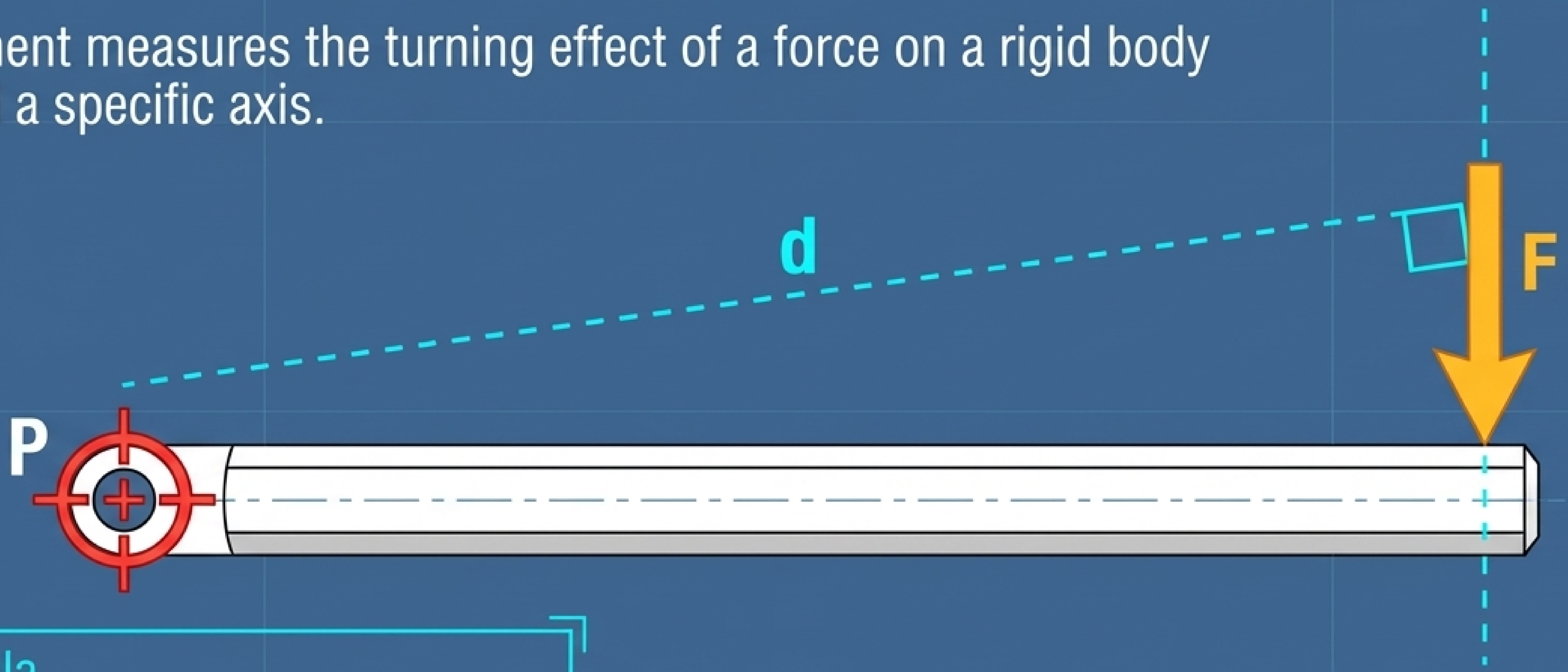
Chapter 8: From basic leverage to the absolute limits of equilibrium.



How does a crane lift 3000kg without snapping or tipping? The answer is the mathematics of balance.

The Core Concept: What is a Moment?

A moment measures the turning effect of a force on a rigid body around a specific axis.



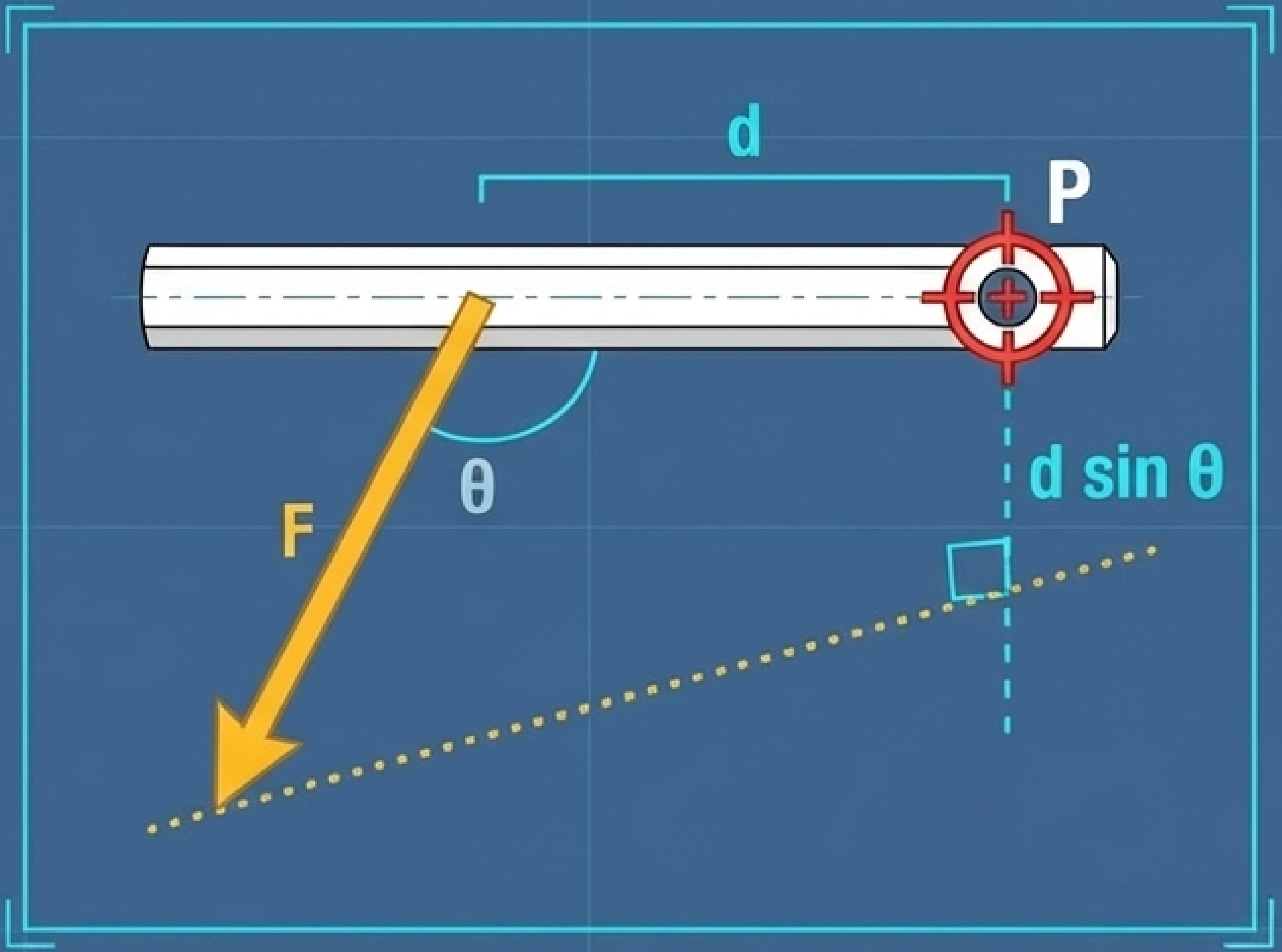
Formula

$$M = |F| \times d$$

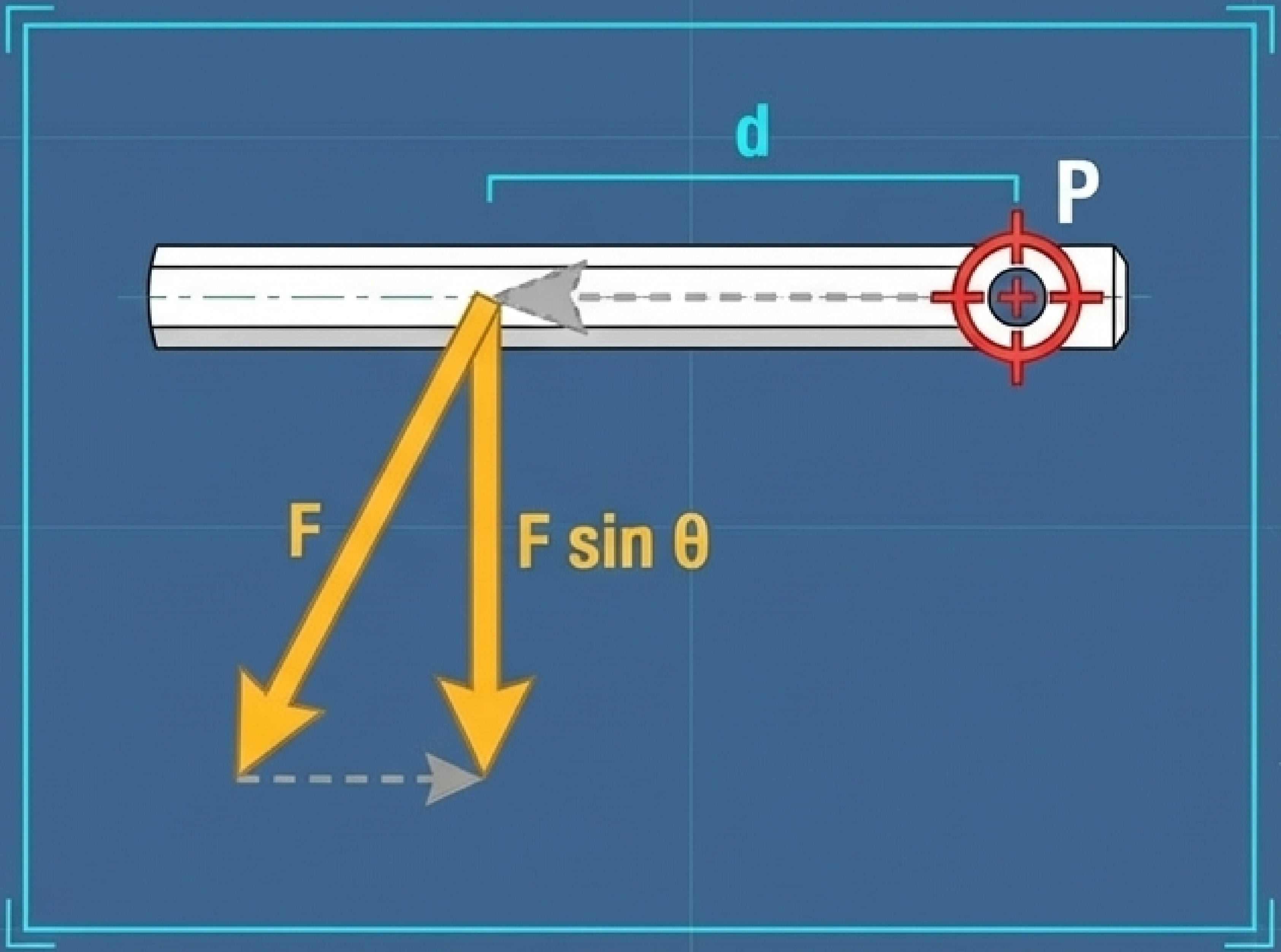
Units: Newton metres (N m)

d MUST be the perpendicular distance from the line of action of the force to the axis of rotation.

The Trigonometry of Moments



Extending the line of action to find perpendicular distance.

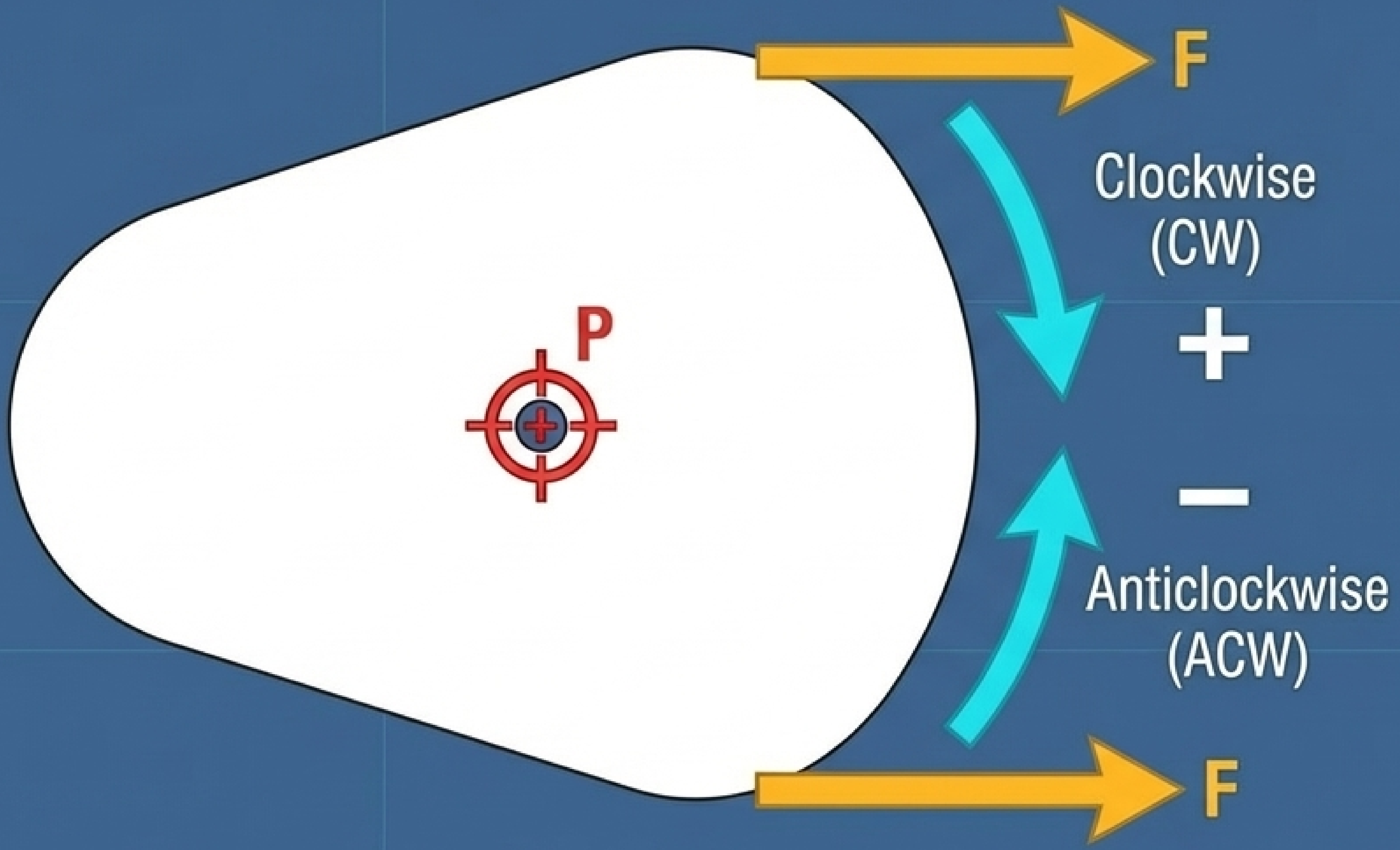


Resolving the force into perpendicular components.

The mathematical outcome is identical: Moment = $F \times d \sin \theta$

Direction & Sign Conventions

Moments are vector quantities. They possess both magnitude and direction.



Convention

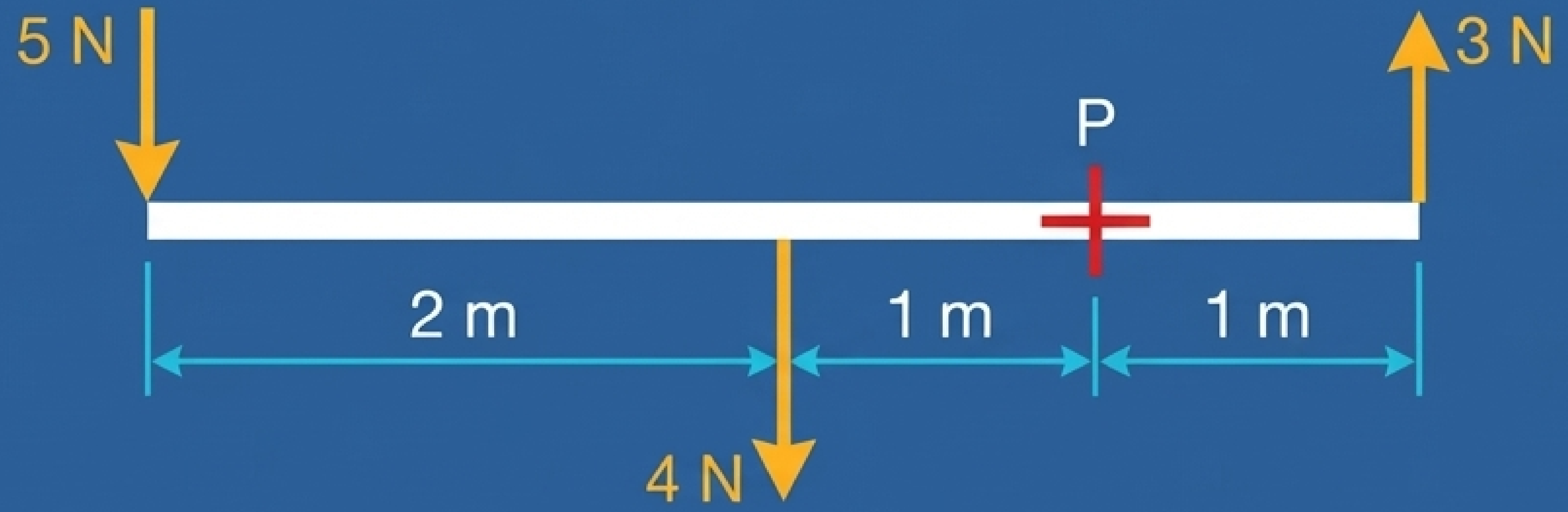
When calculating, you must declare a positive direction.

- Clockwise (CW)
- Anticlockwise (ACW)

Standard practice: Define the dominant rotational direction as positive. Opposing moments become negative.

Resultant Moments: The Sum of All Turns

The Resultant Moment is the net turning effect when several coplanar forces act on a body simultaneously.

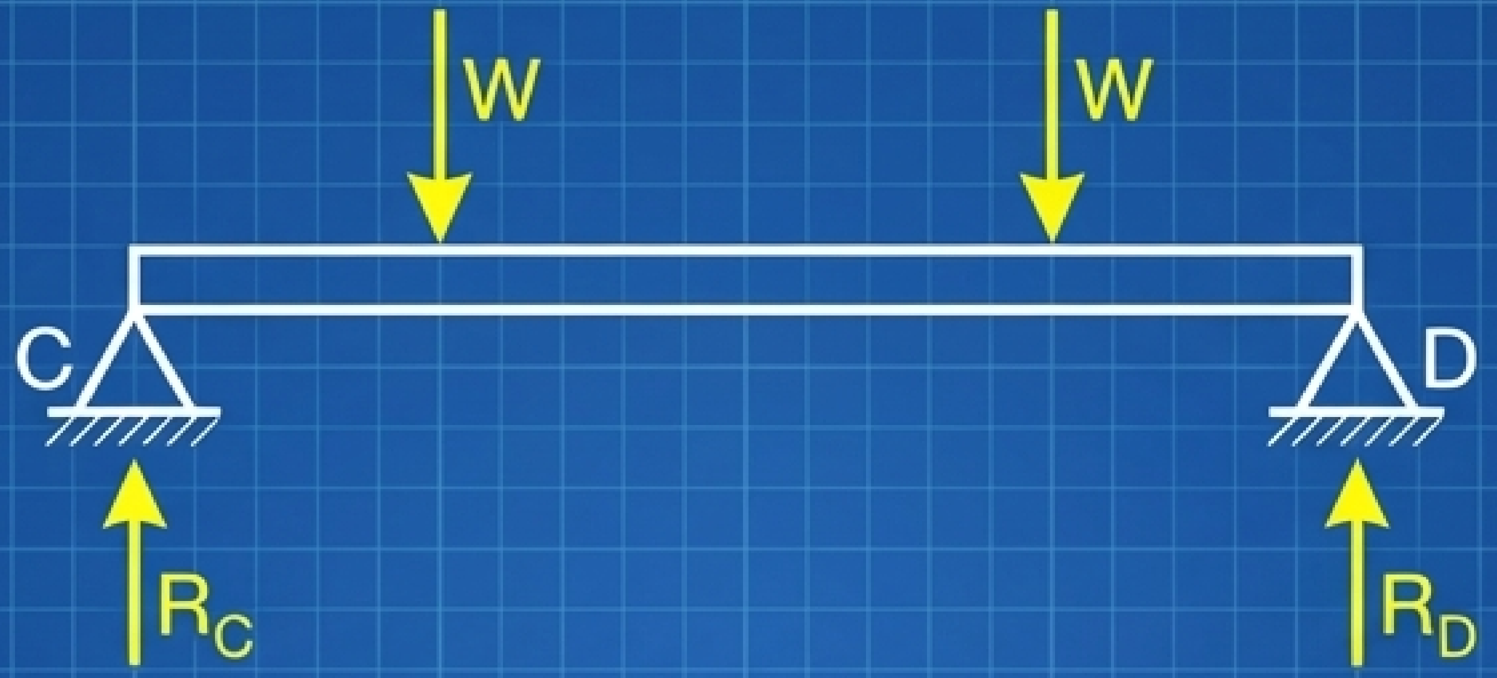


Let Clockwise = Positive (+)

$\sum M = \sum (\text{Clockwise Moments}) - \sum (\text{Anticlockwise Moments})$

If $\sum M > 0$, the body turns clockwise. If $\sum M < 0$, it turns anticlockwise.

The Dual Condition of Static Equilibrium



$\sum F = 0$ (Linear Balance)

Resultant Force is Zero.

Total Upward Forces = Total Downward Forces.

$\sum M = 0$ (Rotational Balance)

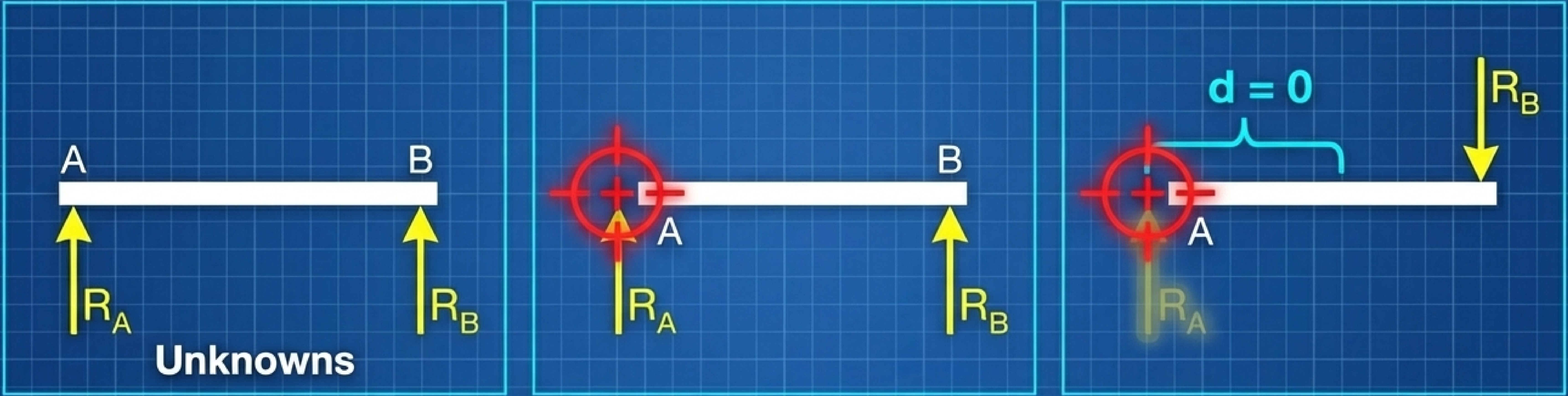
Resultant Moment is Zero.

Total Clockwise Moments = Total Anticlockwise Moments around ANY point.

A rigid body must satisfy both conditions to remain stationary.

The Elimination Trick: Strategic Pivot Selection

Because a body in equilibrium has zero turning effect around any point, you can place your pivot exactly where an unknown force acts.



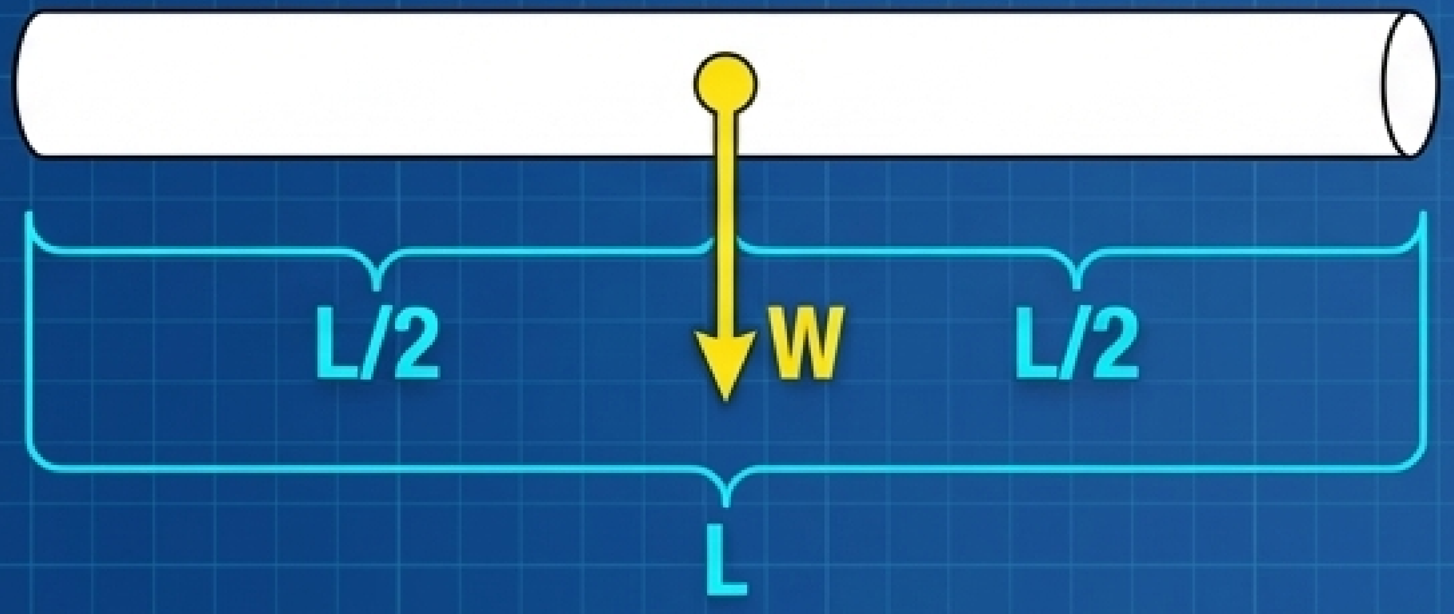
The Math: Moment = Force × 0 meters = 0.

Takeaway: The unknown force vanishes from your equation, allowing you to solve directly for the remaining variables.

Modeling Assumptions: Locating the Weight

in DIN Alternate

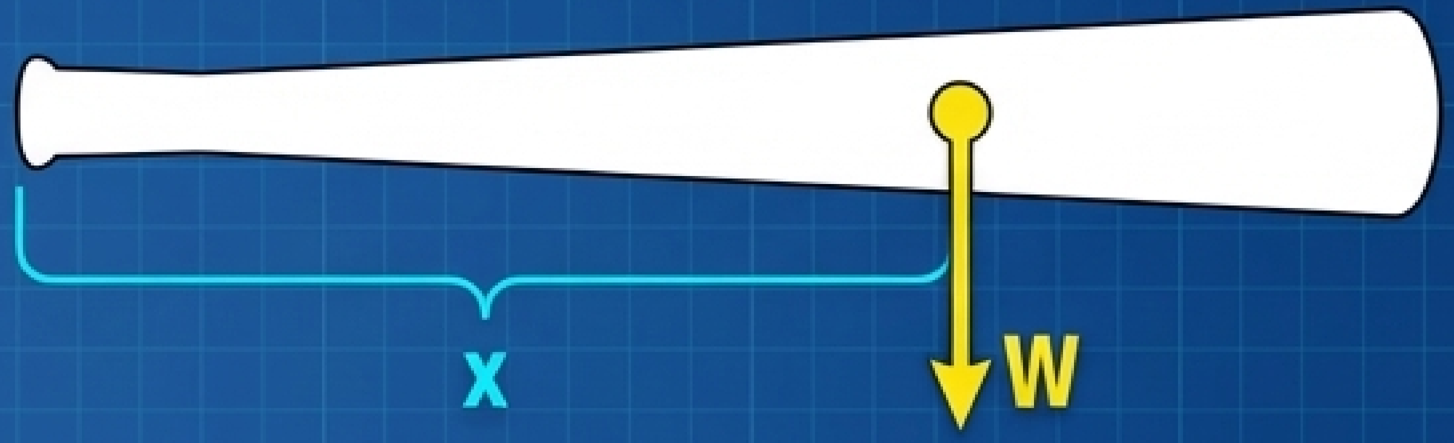
UNIFORM



Physical Reality: Mass is distributed perfectly evenly.

Math Setup: Weight acts exactly at the midpoint. (Distance = Length / 2).

NON-UNIFORM

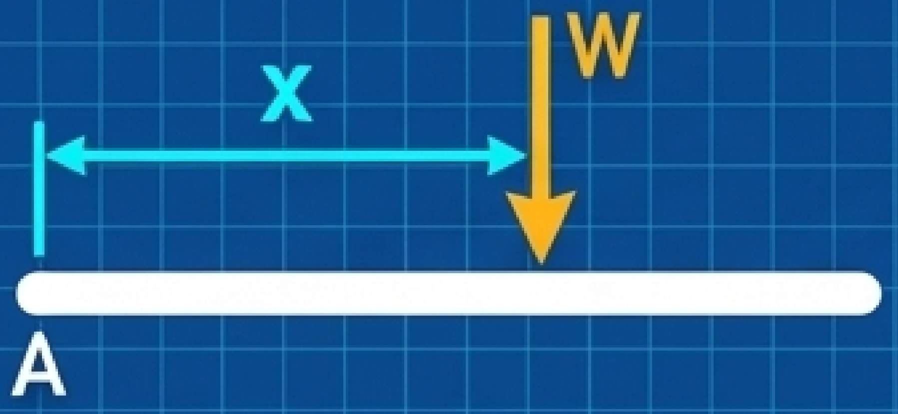


Physical Reality: Mass is unevenly distributed.

Math Setup: Weight acts at an unknown variable distance ‘x’ from an end. ‘x’ must be calculated.

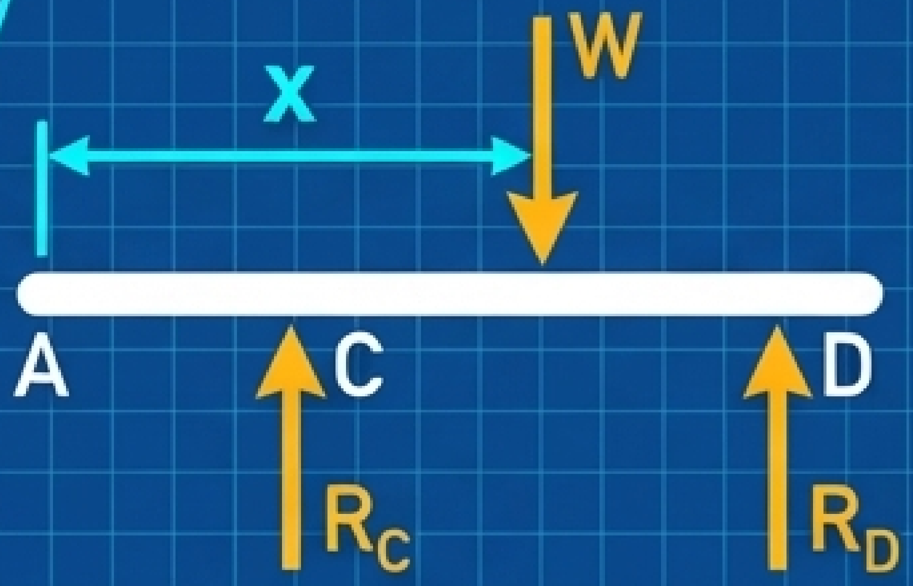
Framework: Finding the Centre of Mass

Step 1: Model & Draw



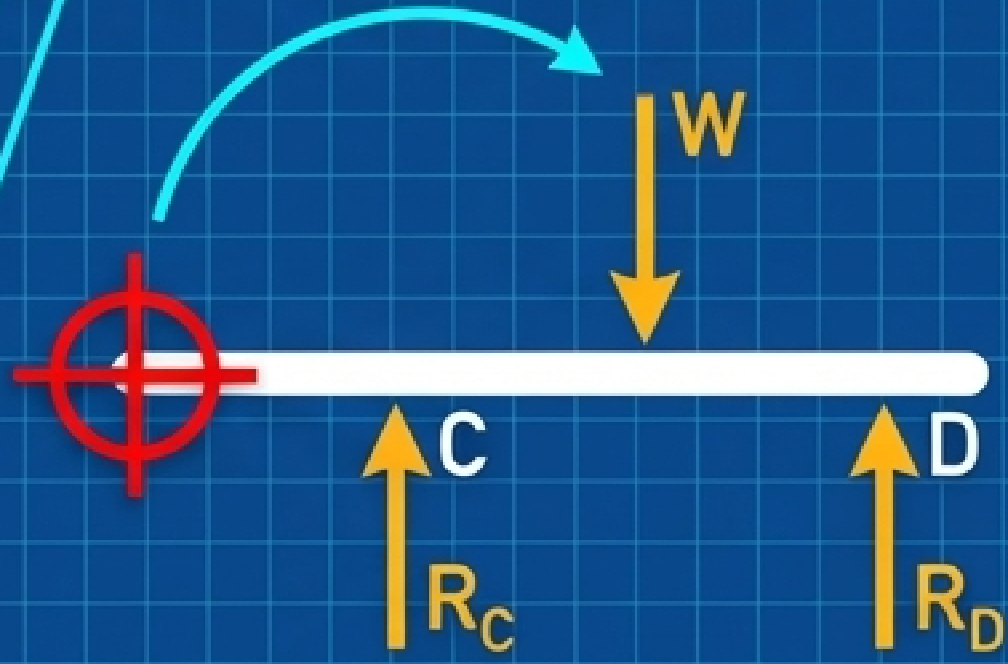
Draw the weight vector **W** at an unknown distance **x** from a designated end.

Step 2: Resolve Vertically



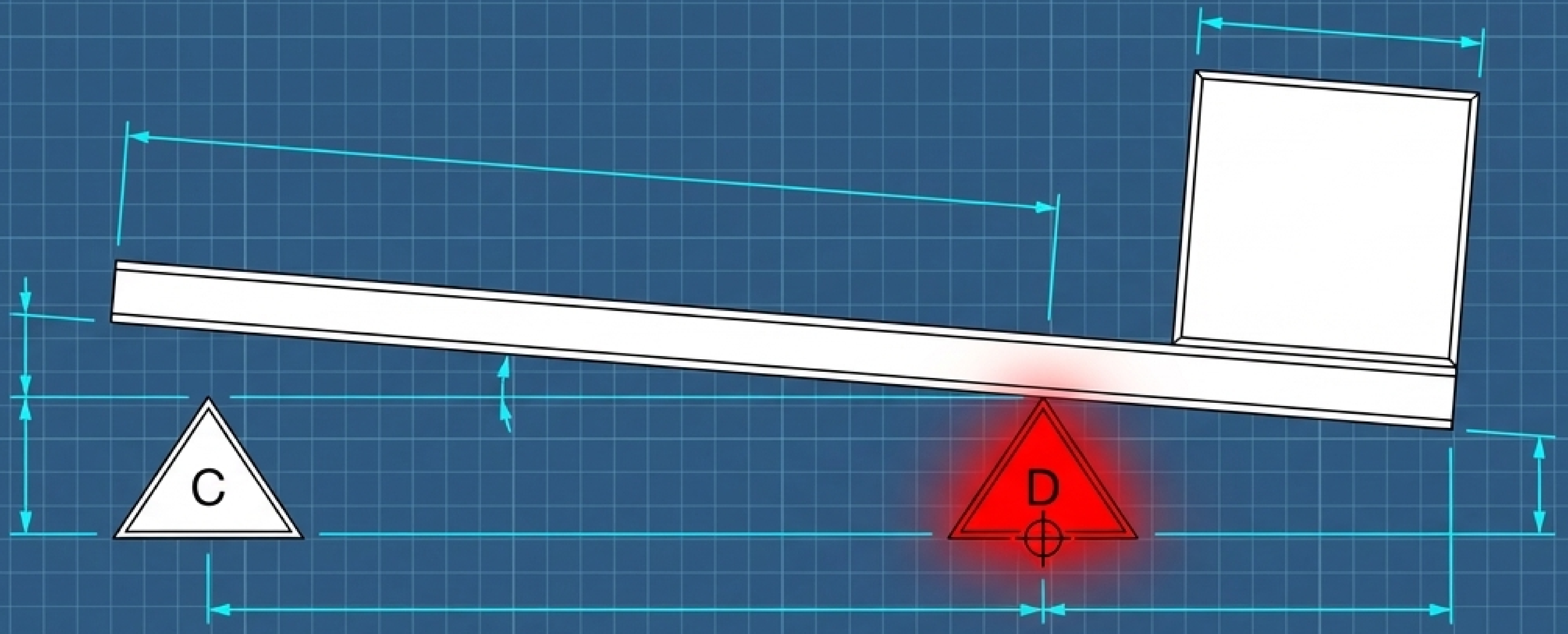
Use $\sum$ **F = 0** (Up = Down) to find any missing reaction forces at the supports.

Step 3: Equate Moments



Take moments about the designated end. Since the system is in equilibrium, CW = ACW. Solve algebraically for x.

The Limit of Balance: Tilting



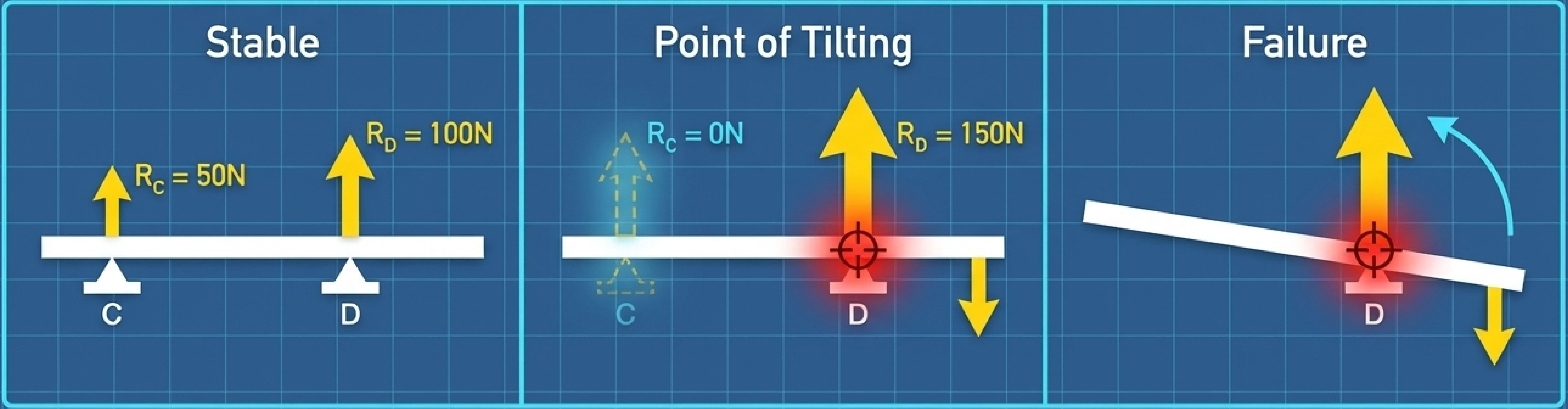
Concept: “On the point of tilting” is the exact mathematical threshold where a body is about to lose equilibrium and rotate.

Physical Reality: The rigid body ceases to act like a beam resting on two supports, and transitions into a lever pivoting entirely around a single point.

The "Vanishing Reaction" Insight

The Golden Rule:

If a rigid body is on the point of tilting about a pivot, the normal reaction at any other support (or the tension in any other wire) is exactly ZERO (R = 0).



Application: Immediately write R = 0 into your diagram when you read "tilting" in an exam prompt.

The Universal Statics Algorithm

1. Map the Blueprint

- Draw a large, clear diagram.
- Add ALL forces (Weights down, Reactions up).
- Add all perpendicular distances.

2. Resolve Linear

- Set Total Forces Up = Total Forces Down.
- ($\Sigma F = 0$).

3. Target the Pivot

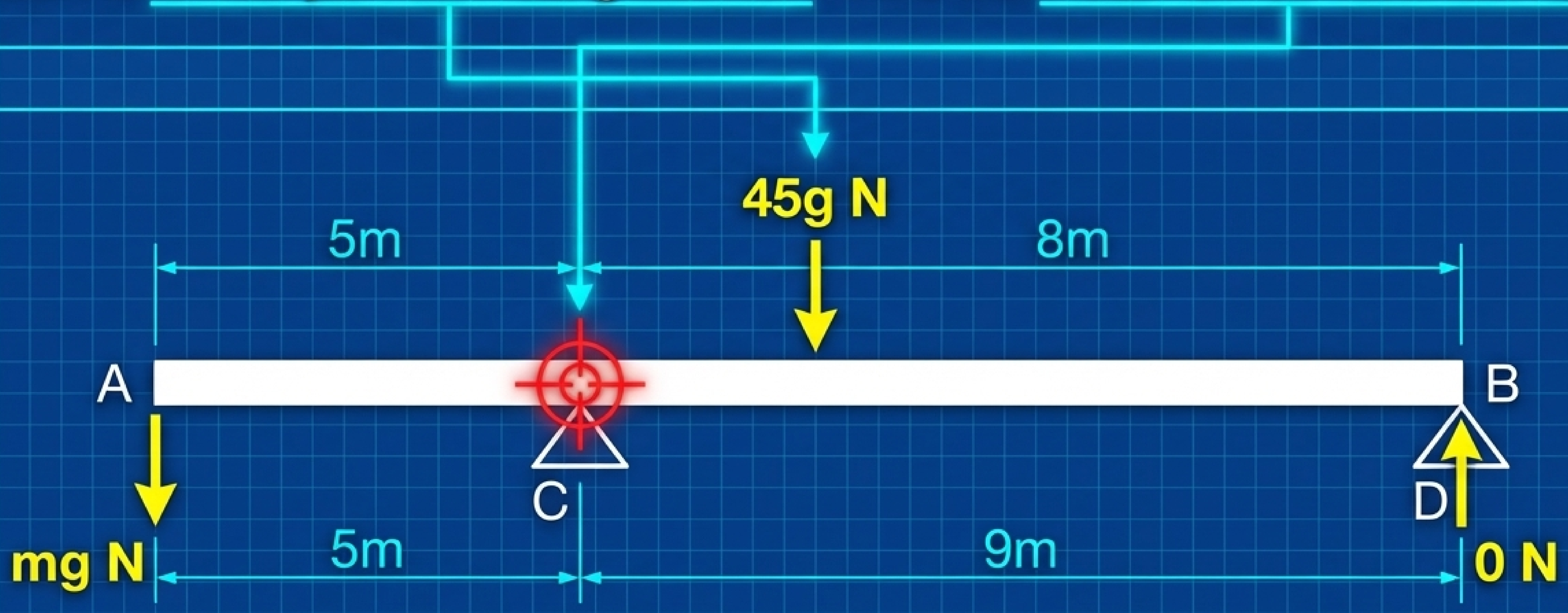
- Place your pivot on an unknown variable to mathematically eliminate it.

4. Equate Rotational

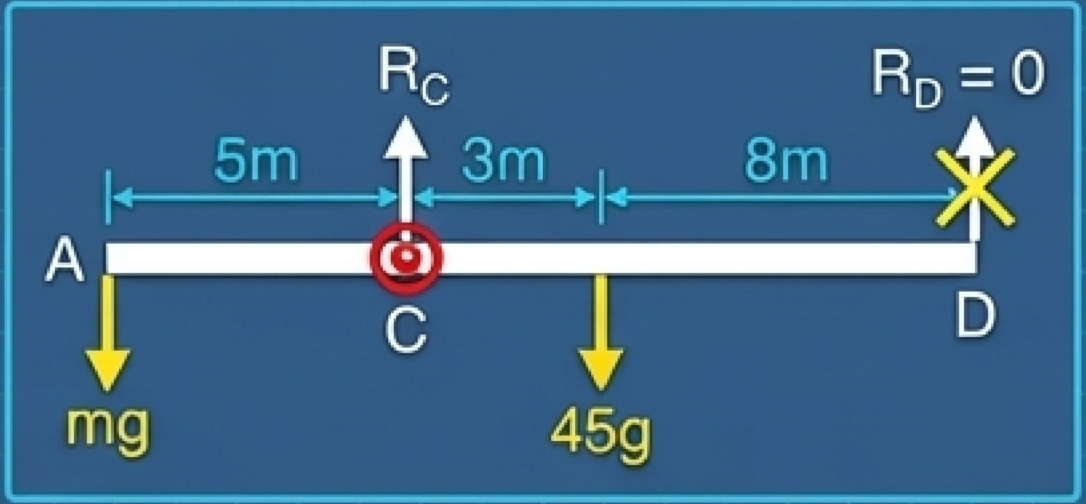
- Set Sum of Clockwise Moments = Sum of Anticlockwise Moments ($\Sigma M = 0$).
- Solve the simultaneous equations.

Masterclass: Deconstructing the Prompt

A **uniform beam AB** of **mass 45kg** and length 16m, rests horizontally on supports C and D where AC = 5m and CD = 9m. When a child stands at A, the beam is **on the point of tilting about C**. Find the **mass of the child is m kg**.



Masterclass: Executing the Solution



Linear Balance

$\sum F = 0$ (Up = Down)

$R_C + R_D = 45g + mg$

Substitute $R_D = 0$ (from tilting rule)

$R_C = 45g + mg$

Rotational Balance

Taking moments about C ($\sum M = 0$)

Anticlockwise = Clockwise

$mg \times 5m = 45g \times 3m$

$5m = 135$

$m = 27$ kg

Because we strategically chose C as the pivot point, the unknown variable R_C was eliminated from the moment equation entirely, completely bypassing the need for simultaneous equations.

The Chapter 8 Blueprint (Cheat Sheet)

Save / Screenshot

Core Math

- **M = F × d** (Perpendicular distance)
- **M = F × d sin θ** (Angled force)
- Units: Nm. Always declare CW or ACW.

Equilibrium

- 1. Resultant Force = 0 N.
- 2. Resultant Moment = 0 Nm.
- Tactic: Place your pivot on an unknown force to eliminate it.

Mass Models

- **Uniform:** Weight acts exactly at the geometric midpoint **(L/2)**.
- **Non-Uniform:** Weight acts at an unknown variable distance **x**.

Tilting Limits

- Point of tilting about **Pivot A** -> Reaction at Support B becomes 0 N.
- The rigid body ceases acting as a supported beam and begins acting as a lever.