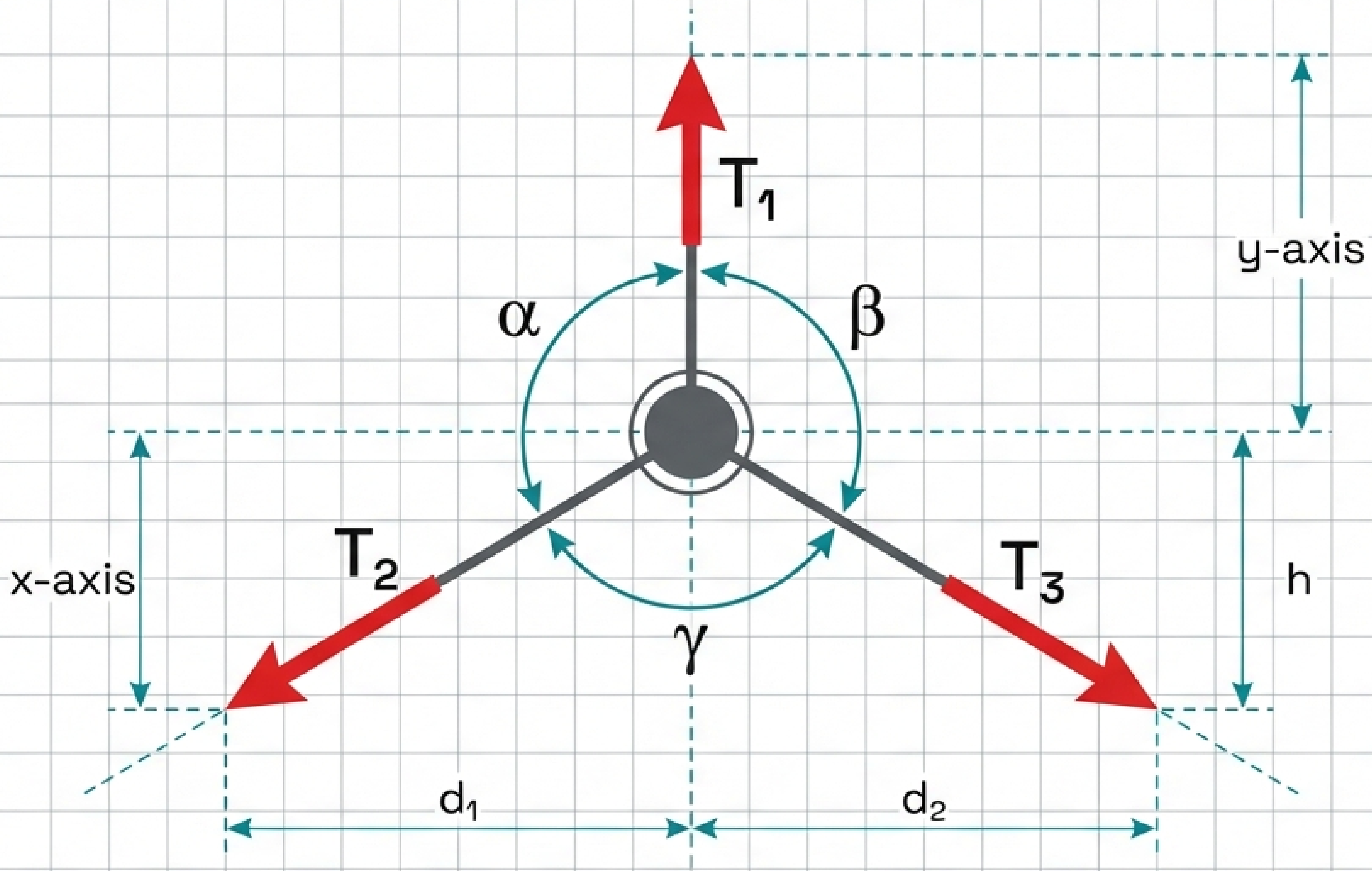
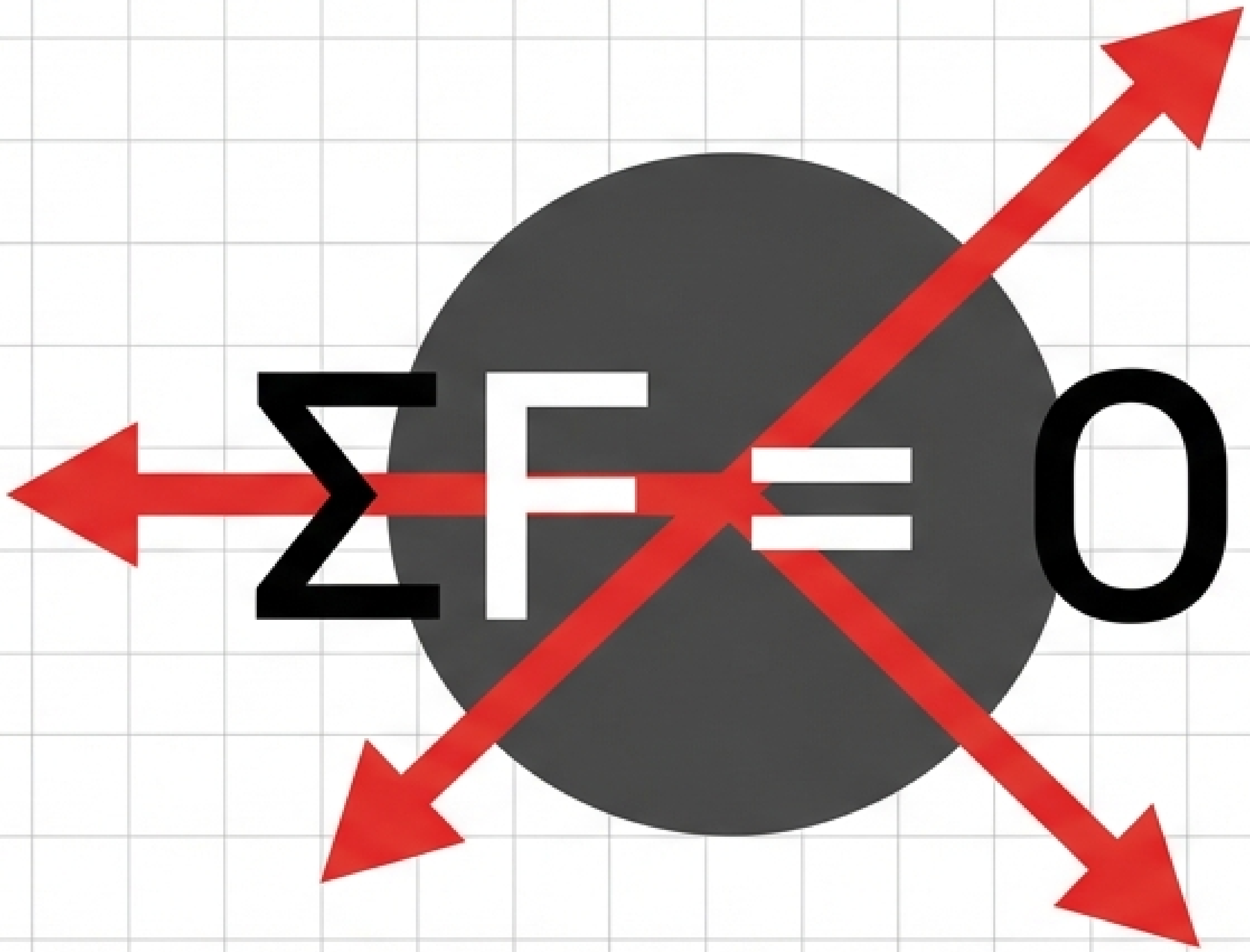
STATICS OF A PARTICLE

The Master Blueprint for Modeling Equilibrium



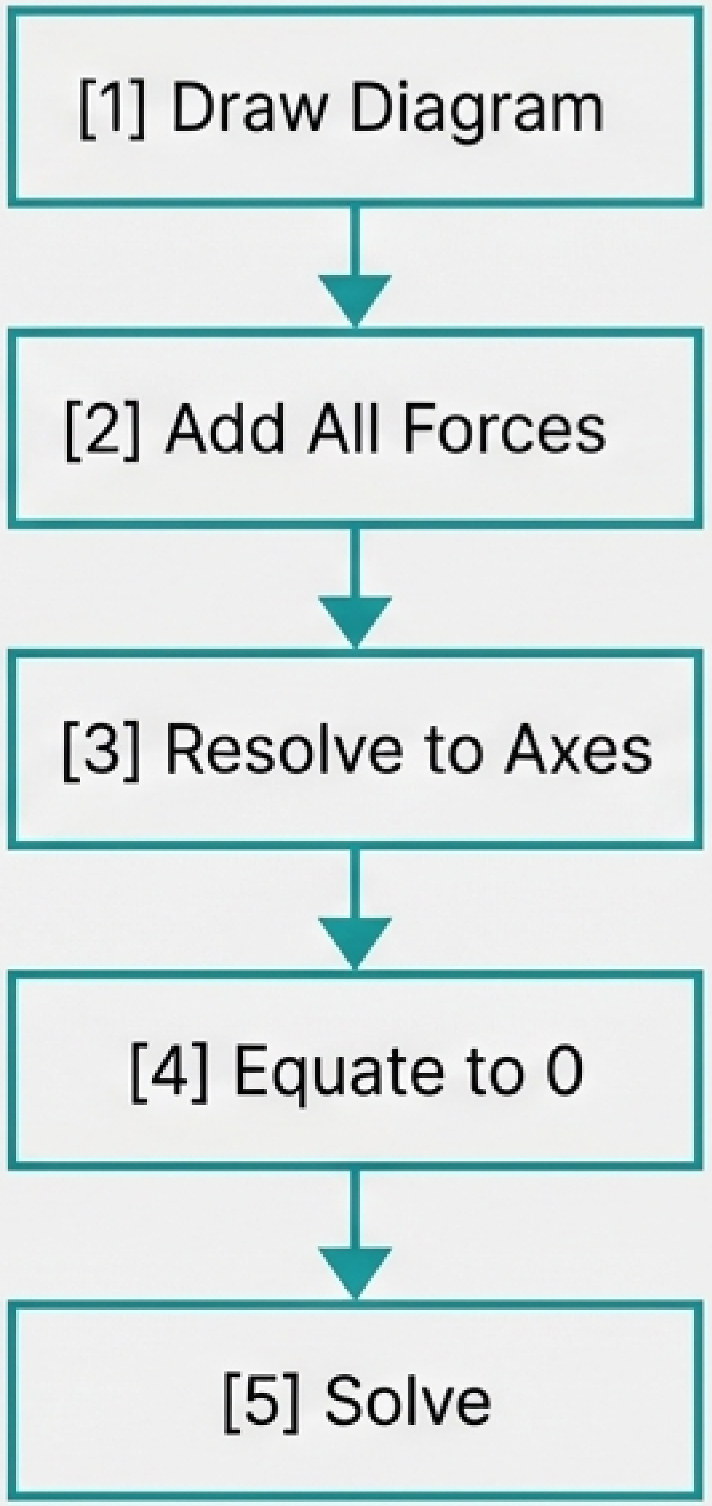
MECHANICS TOOLKIT: CHAPTER 7 SYNTHESIS

The Prime Directive: Resultant Force = 0

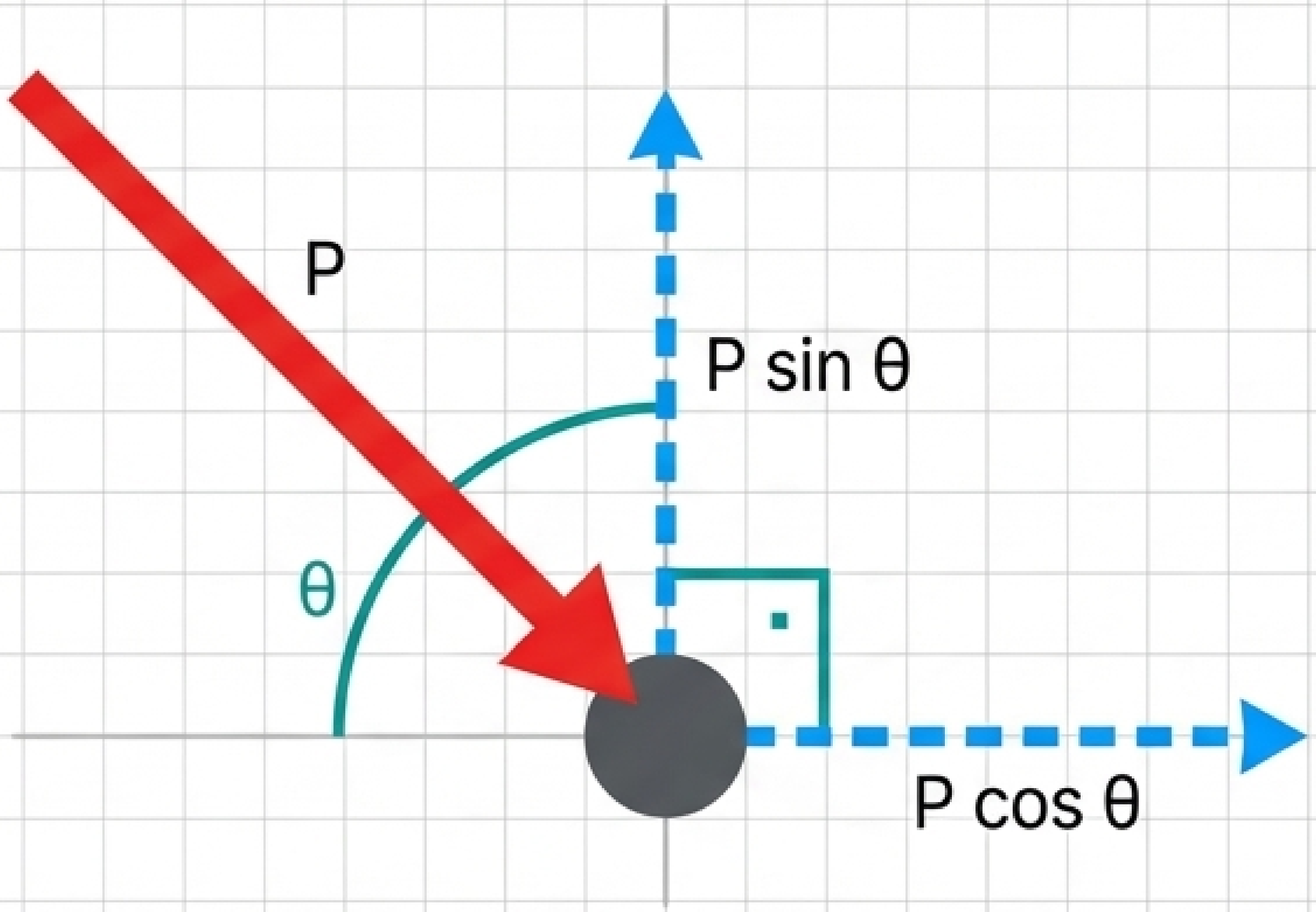


A particle in static equilibrium is completely at rest (or moving at constant velocity). The vector sum of all forces acting upon it is exactly zero.

The Master Algorithm



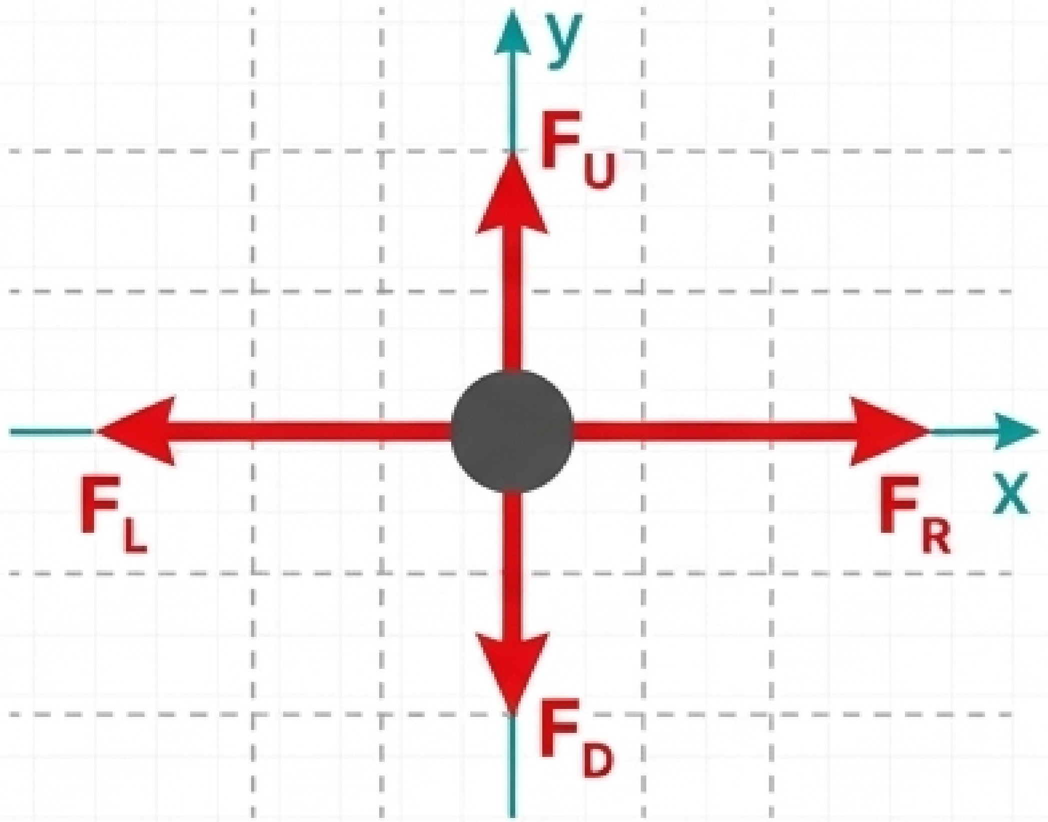
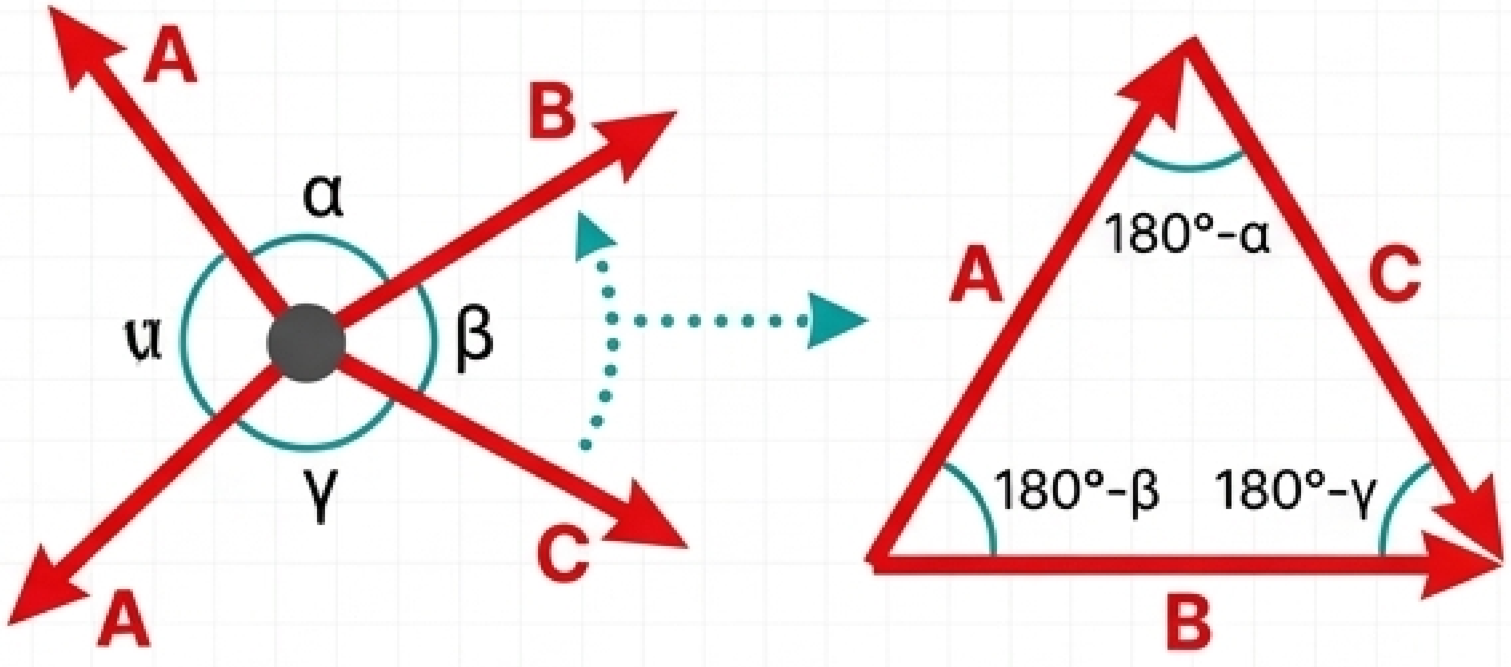
The Toolkit: The Prism Vector Breakdown



Resolving forces replaces one difficult diagonal force with two easy perpendicular forces.

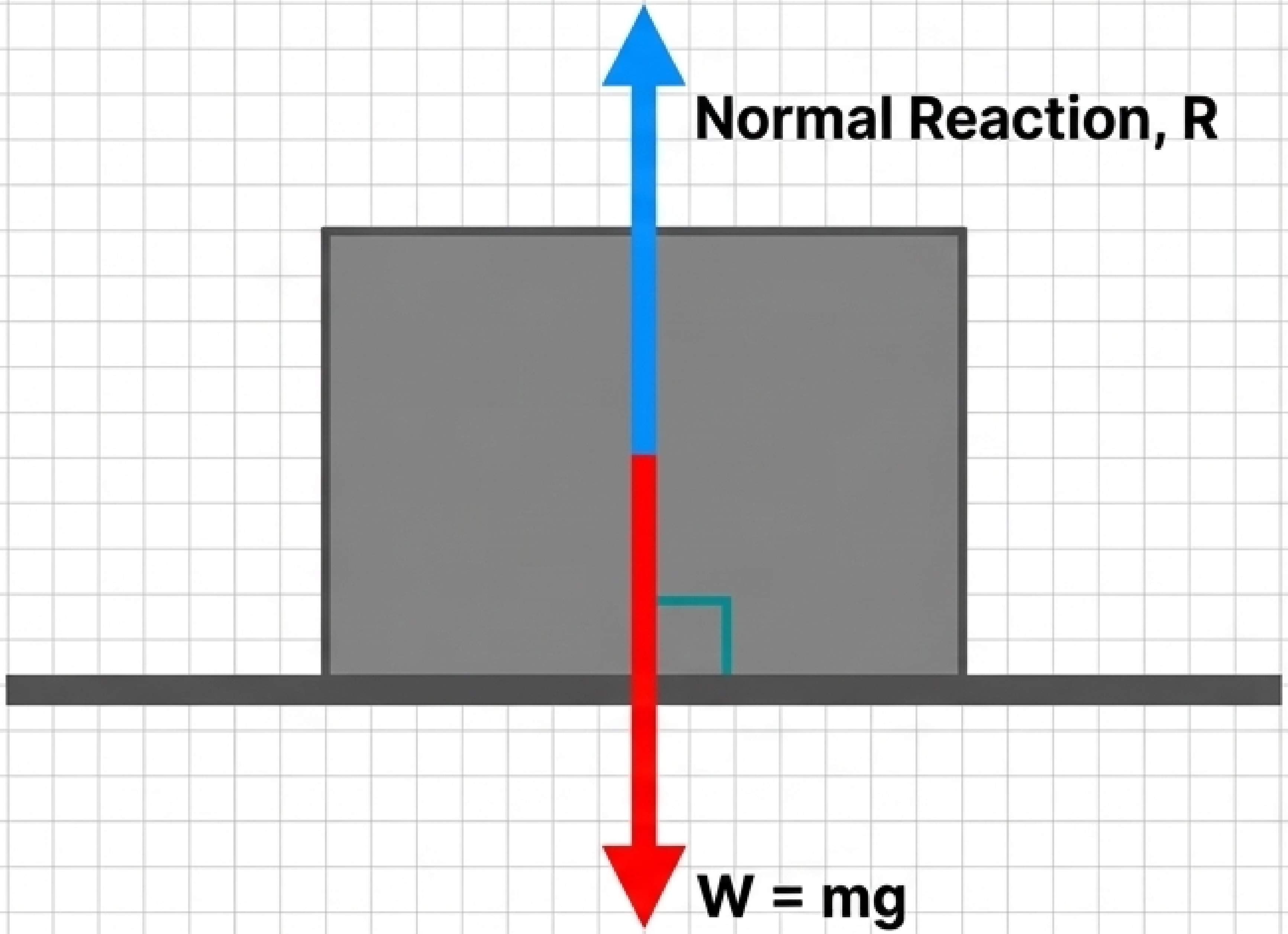
Rule of thumb: 'Cos' closes the angle; 'Sin' slides away.

Phase 1: The Floating Particle (Diagnostic Matrix)

Method 1: Component Resolution (Algebra)	Method 2: Closed Force Triangles (Geometry)
Best for: 4+ forces or unknown angles.	Best for: Exactly 3 forces.
 $\sum F_x = 0$ (Right = Left) $\sum F_y = 0$ (Up = Down)	 Lami's Theorem / Sine Rule: $\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}$

Diagnostic Rule: Choose Resolution for systematic reliability; choose Triangles for pure speed when exactly 3 forces exist.

Phase 2: Grounding the Particle



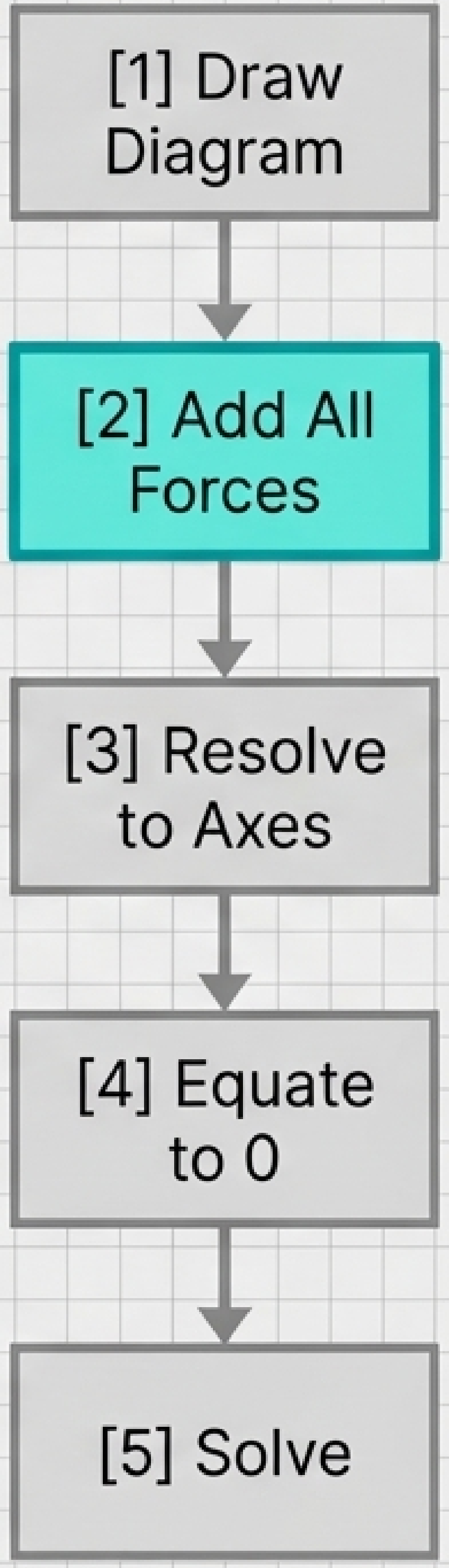
The Weight Vector

Gravity is constant.
Weight always acts strictly vertically downward.

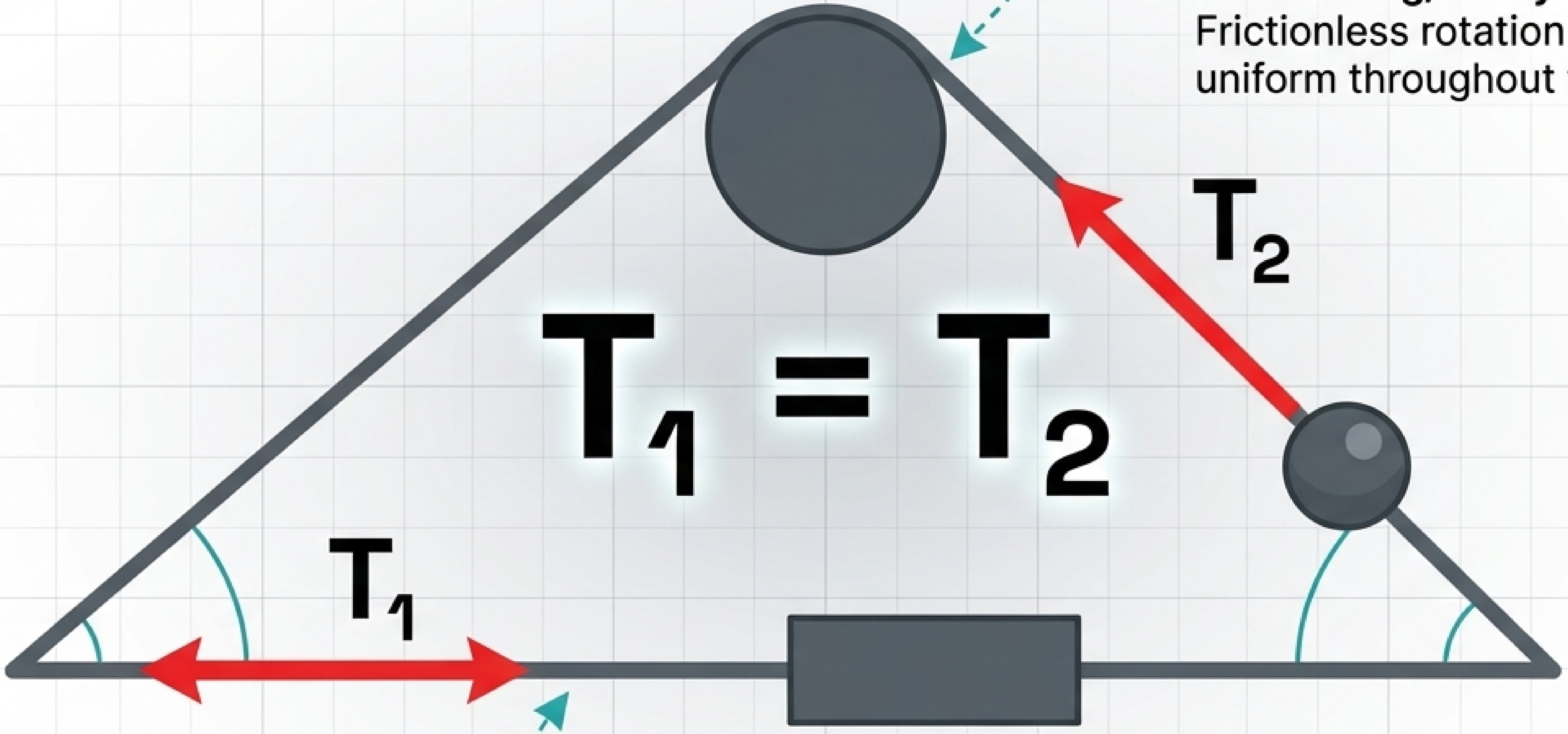
$W = mg$ (where $g = 9.8\ ms^{-2}$).

The Normal Reaction Vector

Surfaces push back. R acts precisely perpendicular (at 90°) to the surface of contact. It only exists as long as contact is maintained.



Phase 2.5: Connected Particles & Tension



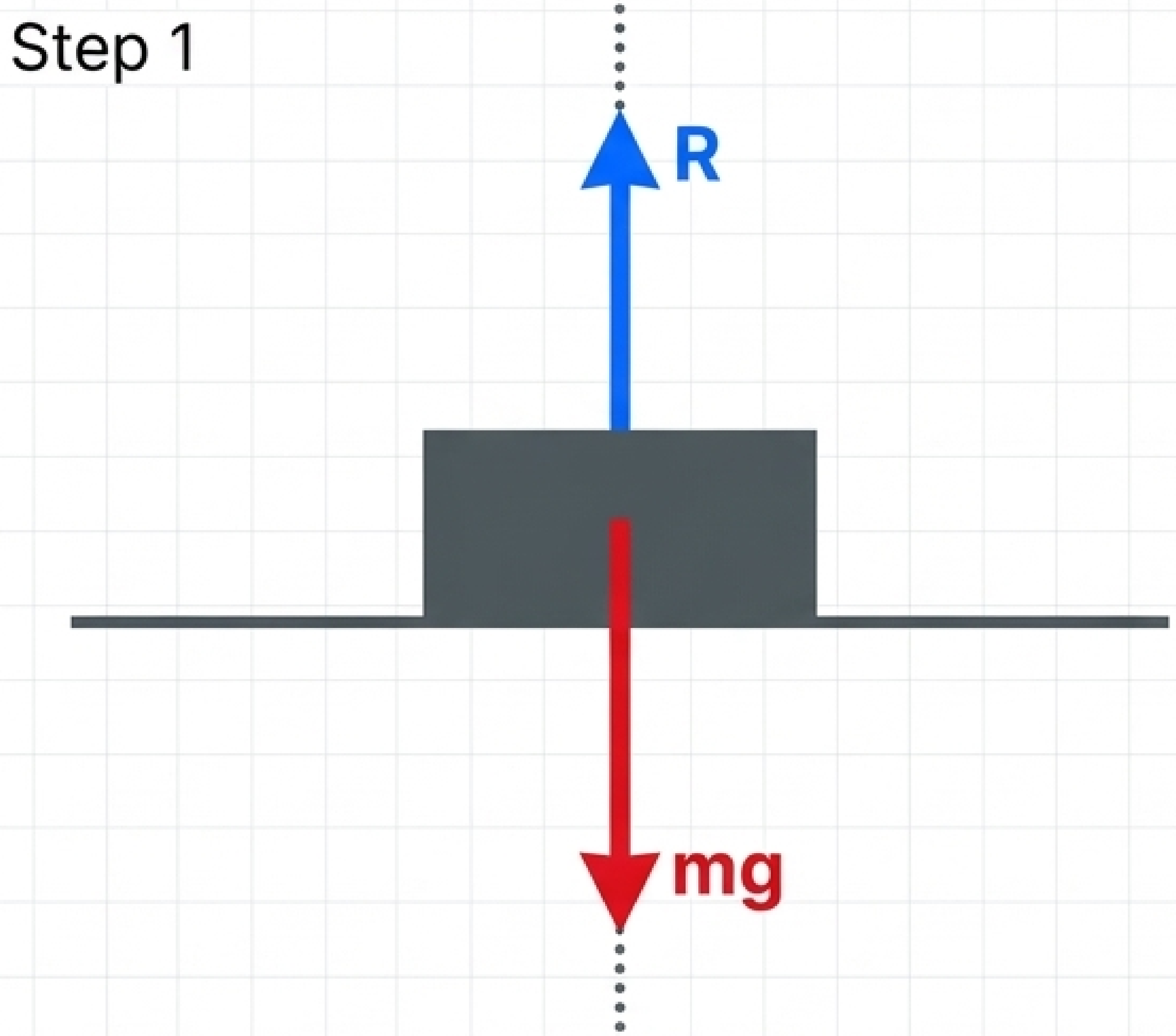
Smooth Peg/Pulley assumption: Frictionless rotation means Tension is uniform throughout the entire string.

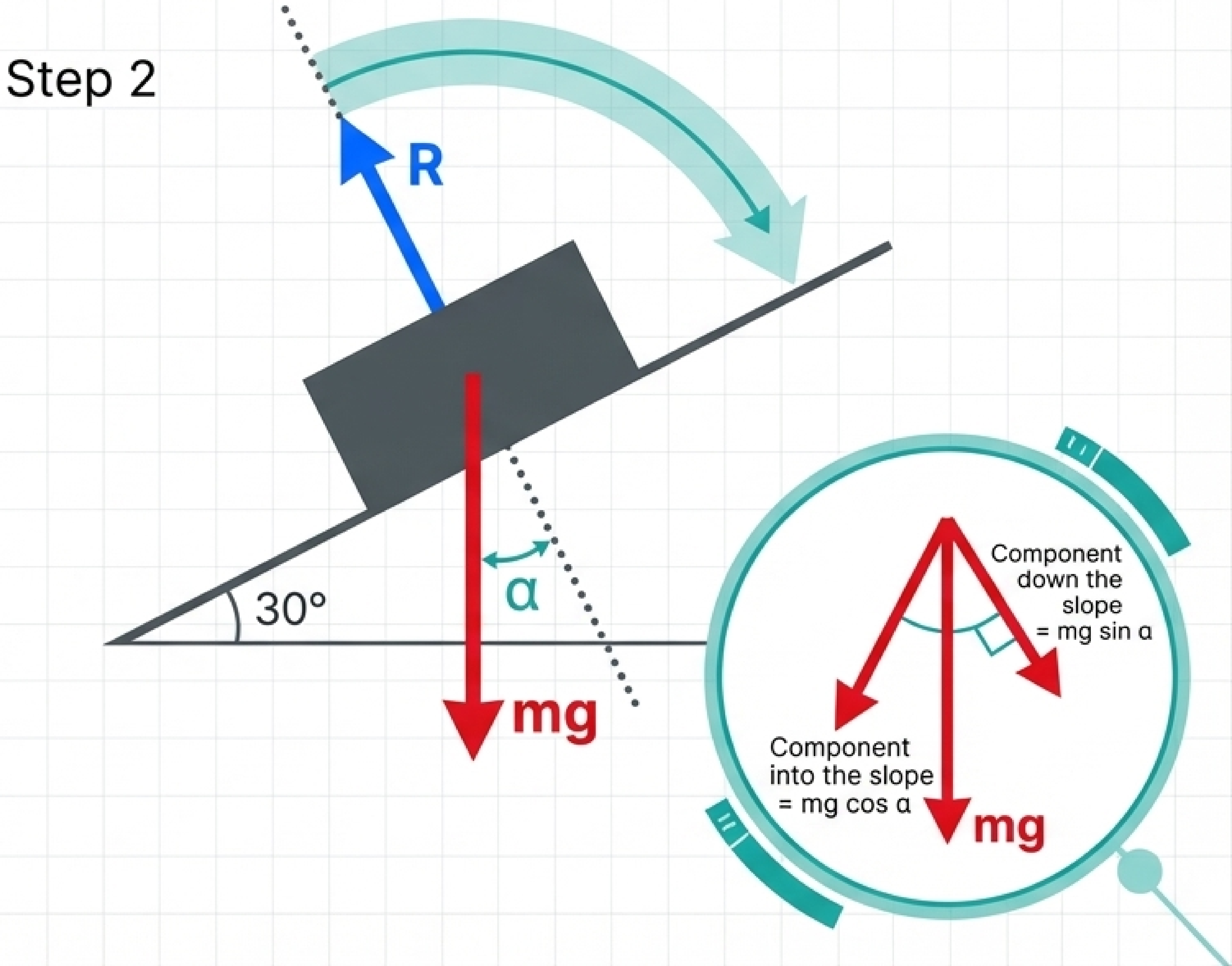
Light String assumption: Zero mass means it is ignored in force calculations.

Inextensible assumption: Cannot stretch. Both masses share identical acceleration or remain in perfect static lockstep.

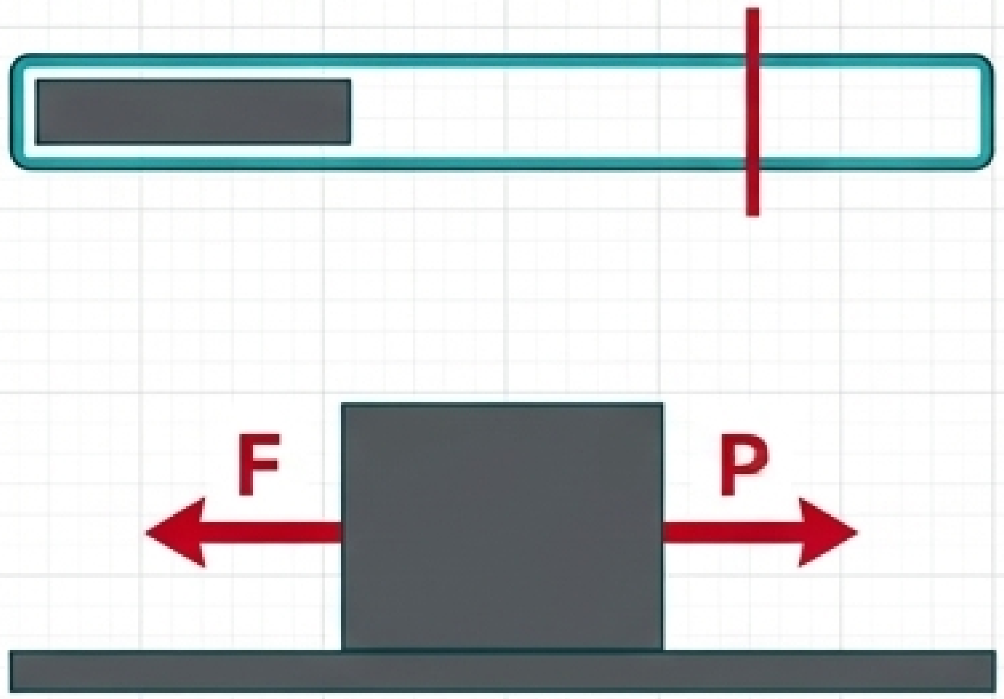
Phase 3: The Tilted World

The Gravity Rotation Sequence



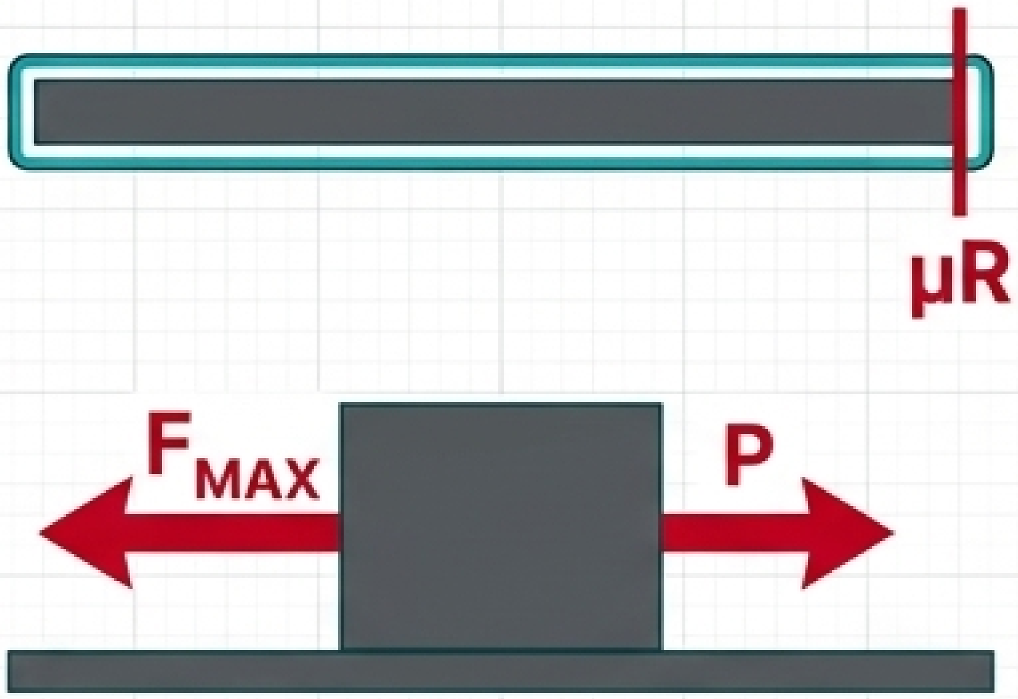


Phase 4: The Friction Bank ($F \leq \mu R$)



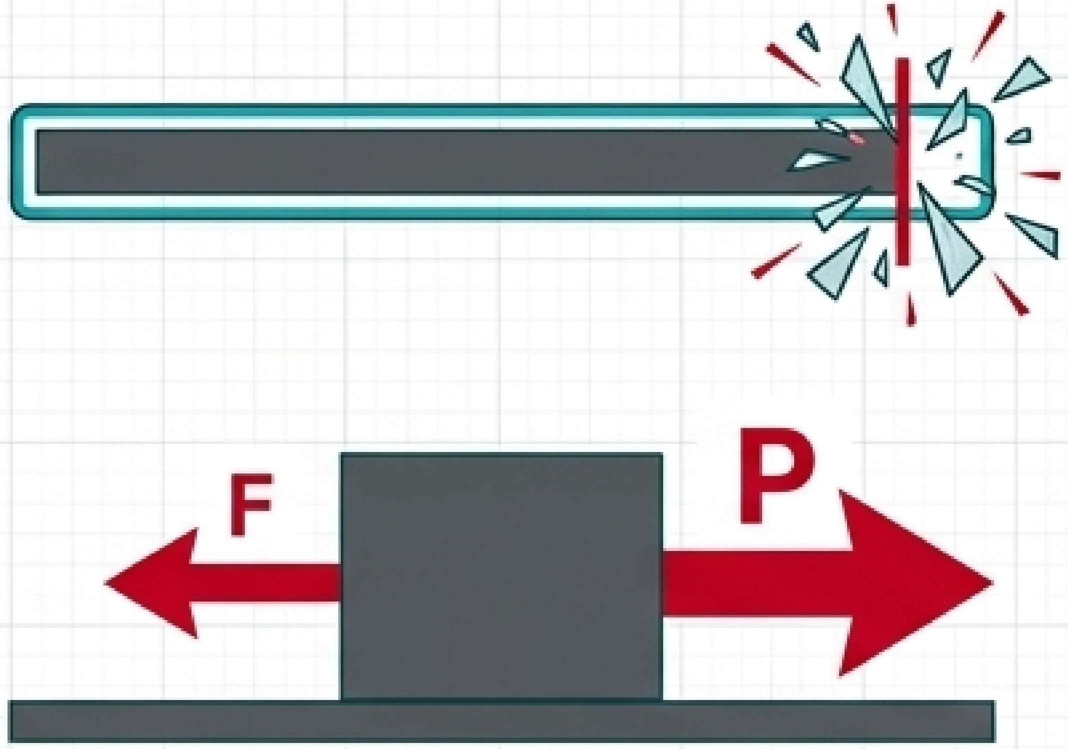
Static State

The friction force simply matches the applied force.
$F = P$



Limiting Equilibrium

The block is on the absolute verge of slipping. The friction bank is maxed out.
$F_{MAX} = \mu R$



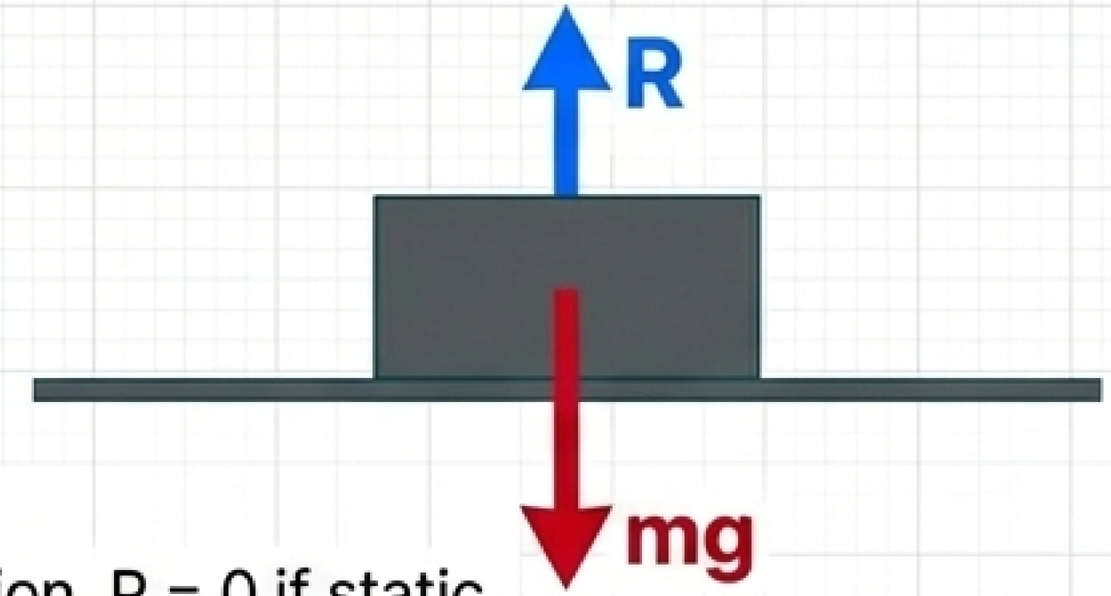
Statics Broken

Applied force exceeds maximum friction. Acceleration begins.

Coefficient of Friction (μ): A dimensionless measure of surface roughness. Smooth surface $\mu = 0$

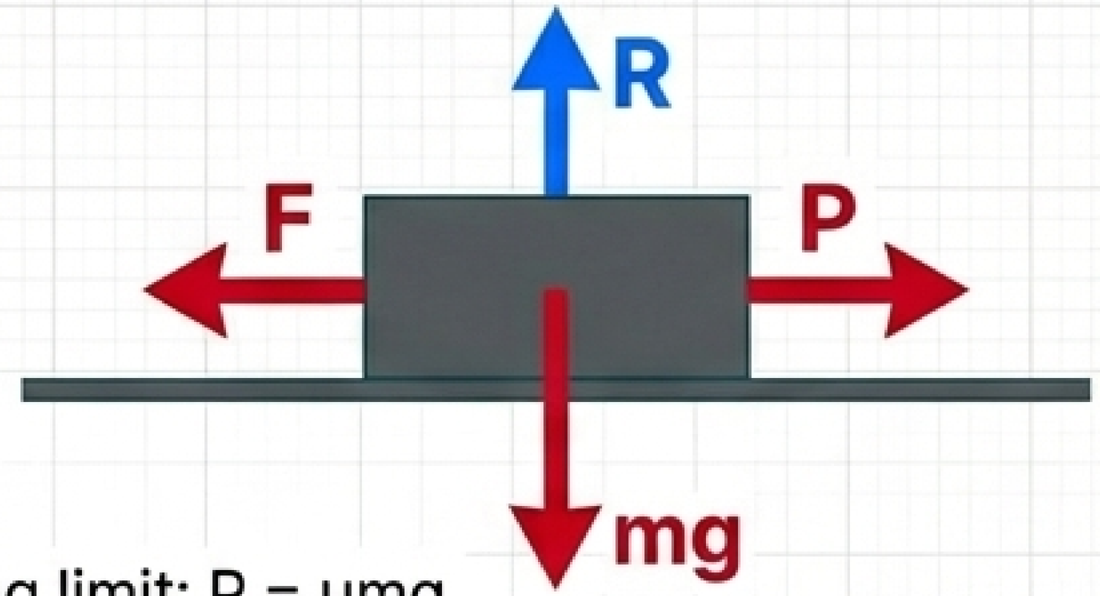
The Surface Matrix: Core Environmental Models

Horizontal + Smooth



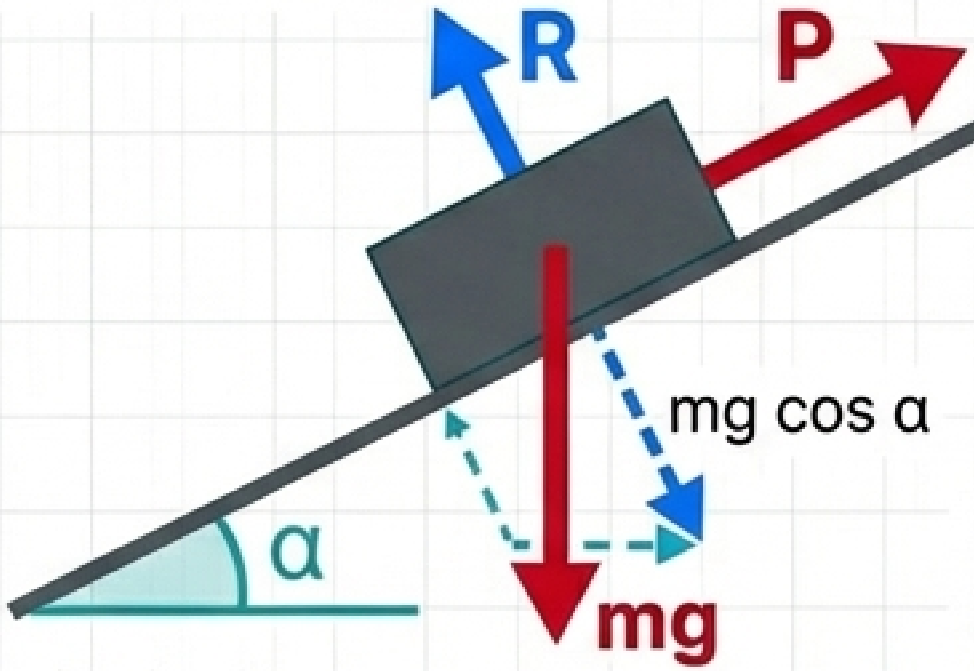
R = mg
No friction. P = 0 if static.

Horizontal + Rough



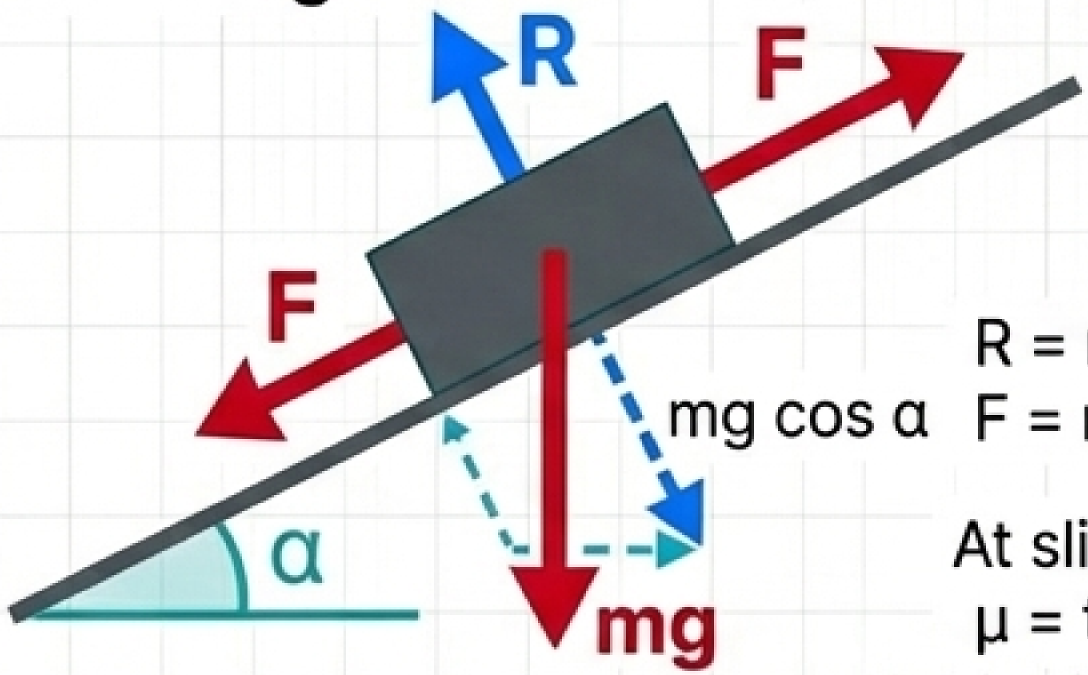
R = mg
P = F
At slipping limit: P = μmg

Inclined + Smooth



R = mg cos α
P = mg sin α

Inclined + Rough

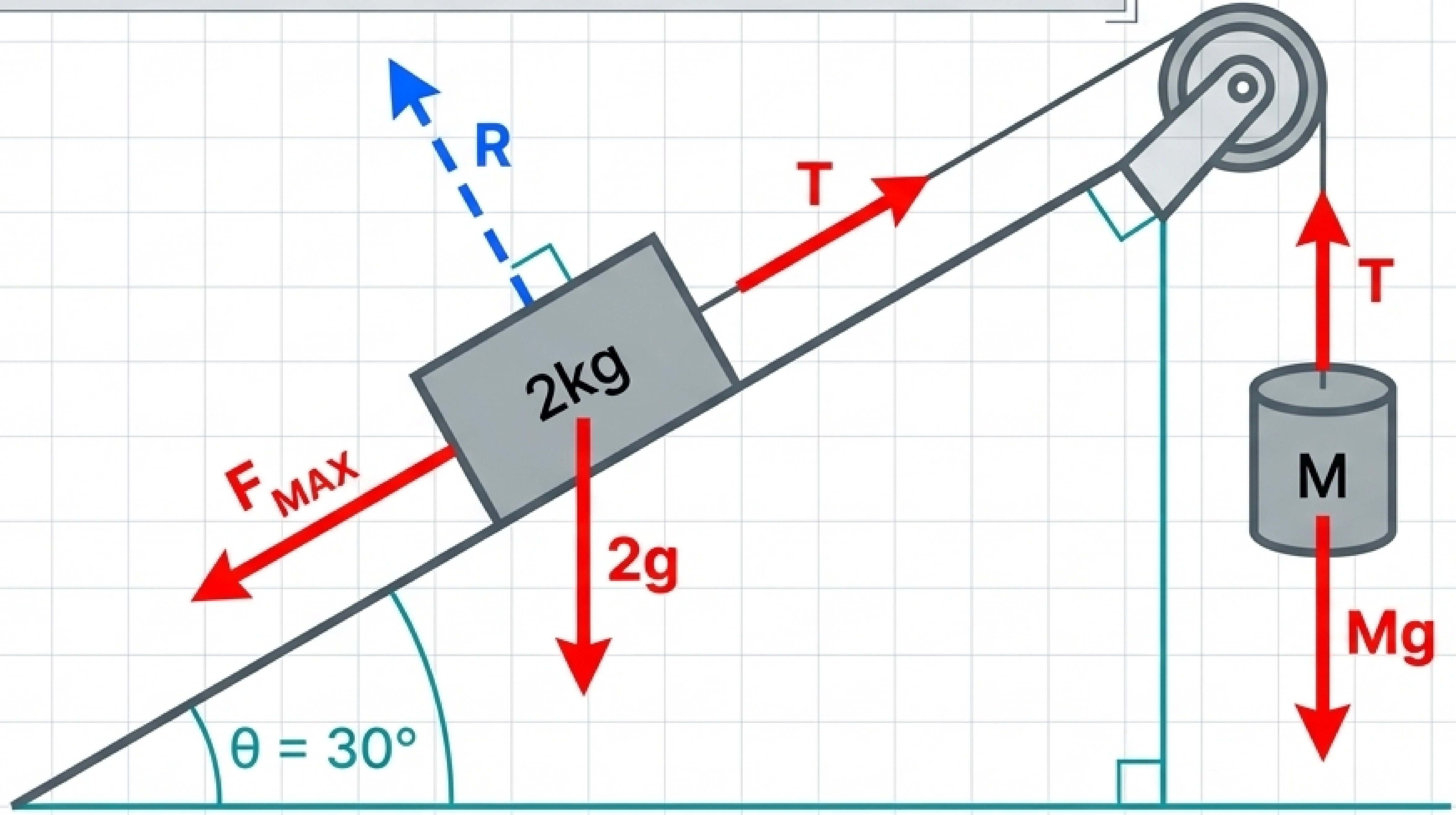


R = mg cos α
F = mg sin α

At slipping limit:
μ = tan α

Phase 5 Synthesis: The Ultimate Static System (Setup)

Scenario: A 2kg mass on a rough inclined plane (θ = 30°) is attached to a string passing over a smooth pulley to a hanging mass M. The system is in limiting equilibrium, on the point of sliding UP the plane. μ = 0.4.



[1] Draw Diagram

[2] Add All Forces

[3] Resolve Forces

[4] Apply Equilibrium Equations (ΣF = 0)

[5] Solve for Unknowns

Phase 5 Synthesis: Solving the Master System

Resolve & Equate Perpendicular (↖)

$$R - 2g\cos 30° = 0$$

R = 16.97 N

The Friction Threshold (The Roughness Link)

Because the block is on the point of sliding, we unlock the limiting formula.

$$F_{MAX} = \mu R \rightarrow 0.4 \times 16.97$$

$$\mathbf{F_{MAX} = 6.79\ N}$$

Resolve & Equate Parallel (↗)

Examine the forces lying directly along the string line.

$$T - F_{MAX} - 2g\sin 30° = 0$$

$$T = 6.79 + 9.8 = \mathbf{16.59\ N}$$

Since T = Mg (hanging mass equilibrium): **M = 16.59 / 9.8 = 1.69 kg**

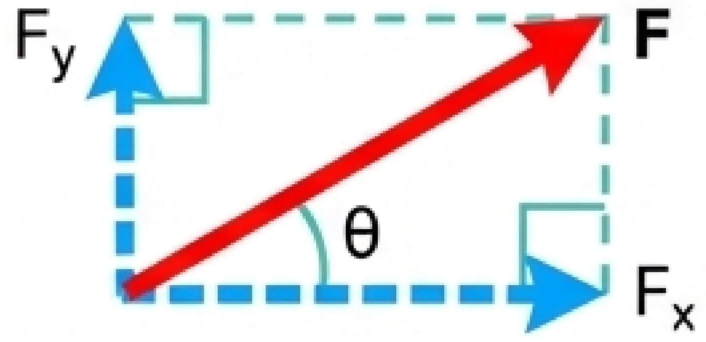
[1] Draw Diagram
[2] Add All Forces
[3] Resolve to Axes
[4] Equate to 0
[5] Solve

The Engineer's Toolkit: Chapter 7 Summary

Equilibrium Prime Law

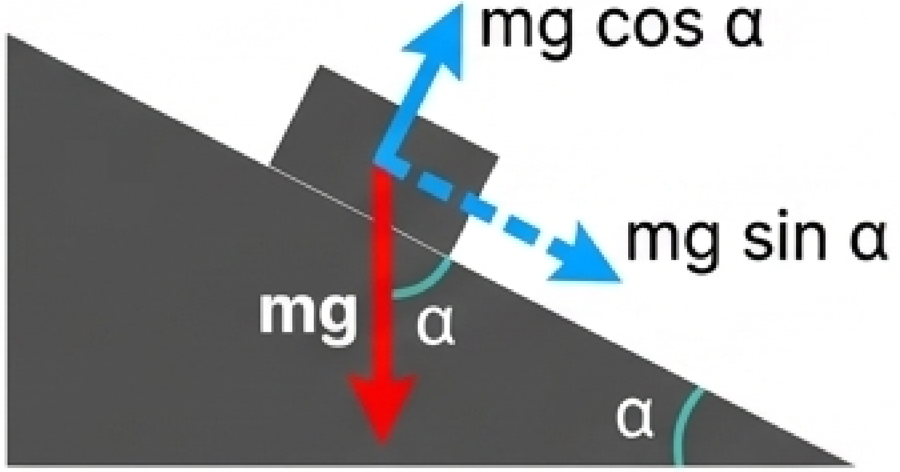
$\sum F_x = 0$ and $\sum F_y = 0$.
All vectors must close the loop.

The Resolution Prism



$F_x = F\cos\theta$. $F_y = F\sin\theta$.
Cosine always closes the angle.

The Gravity Tilt



Normal $R = mg\cos\alpha$. Slide Force $= mg\sin\alpha$.

The Friction Gauge

Static: $F \leq \mu R$. Limiting: $F = \mu R$.
Friction strictly opposes impending motion.

Mechanics 1: Master the model, trust the algorithm, balance the system.