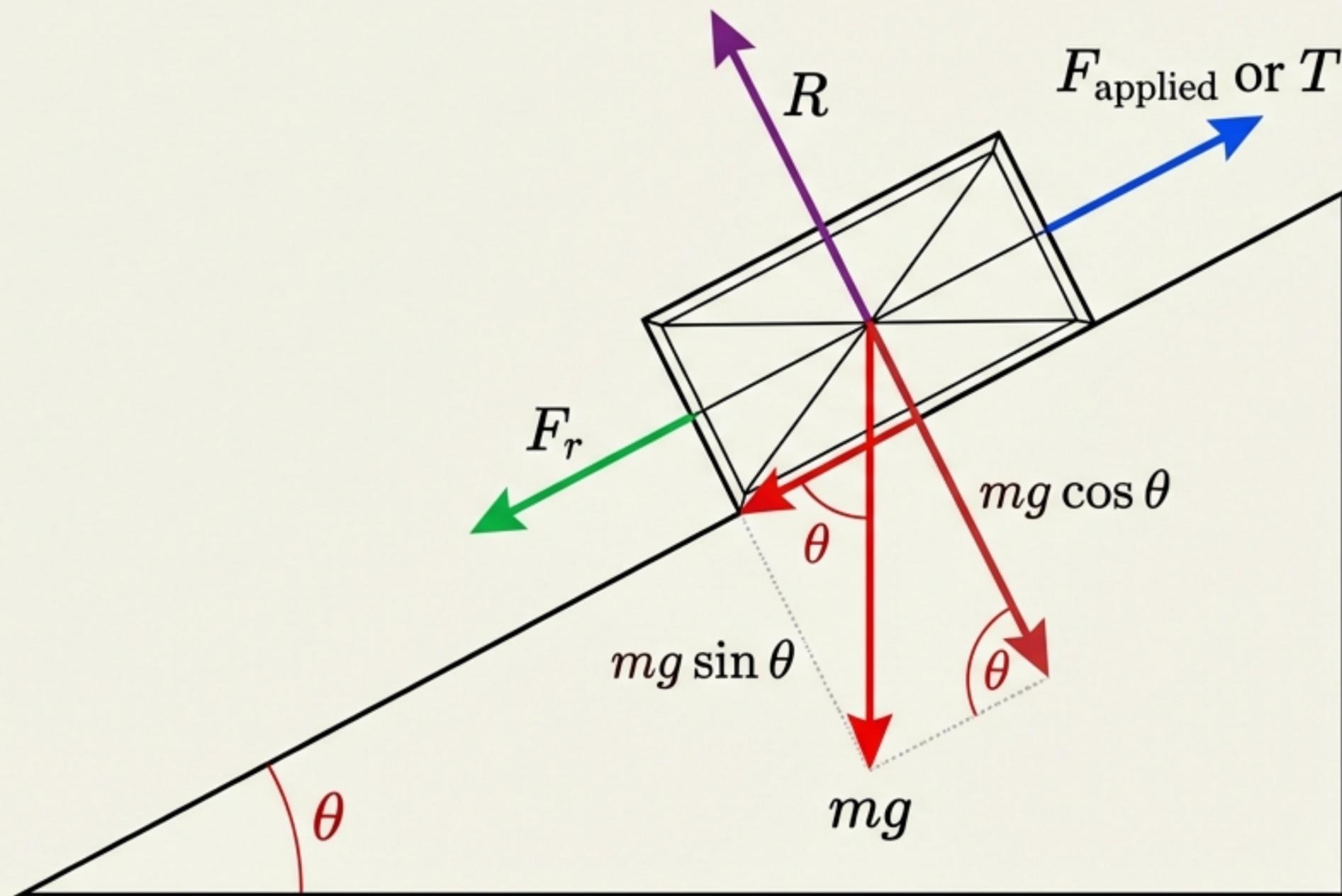


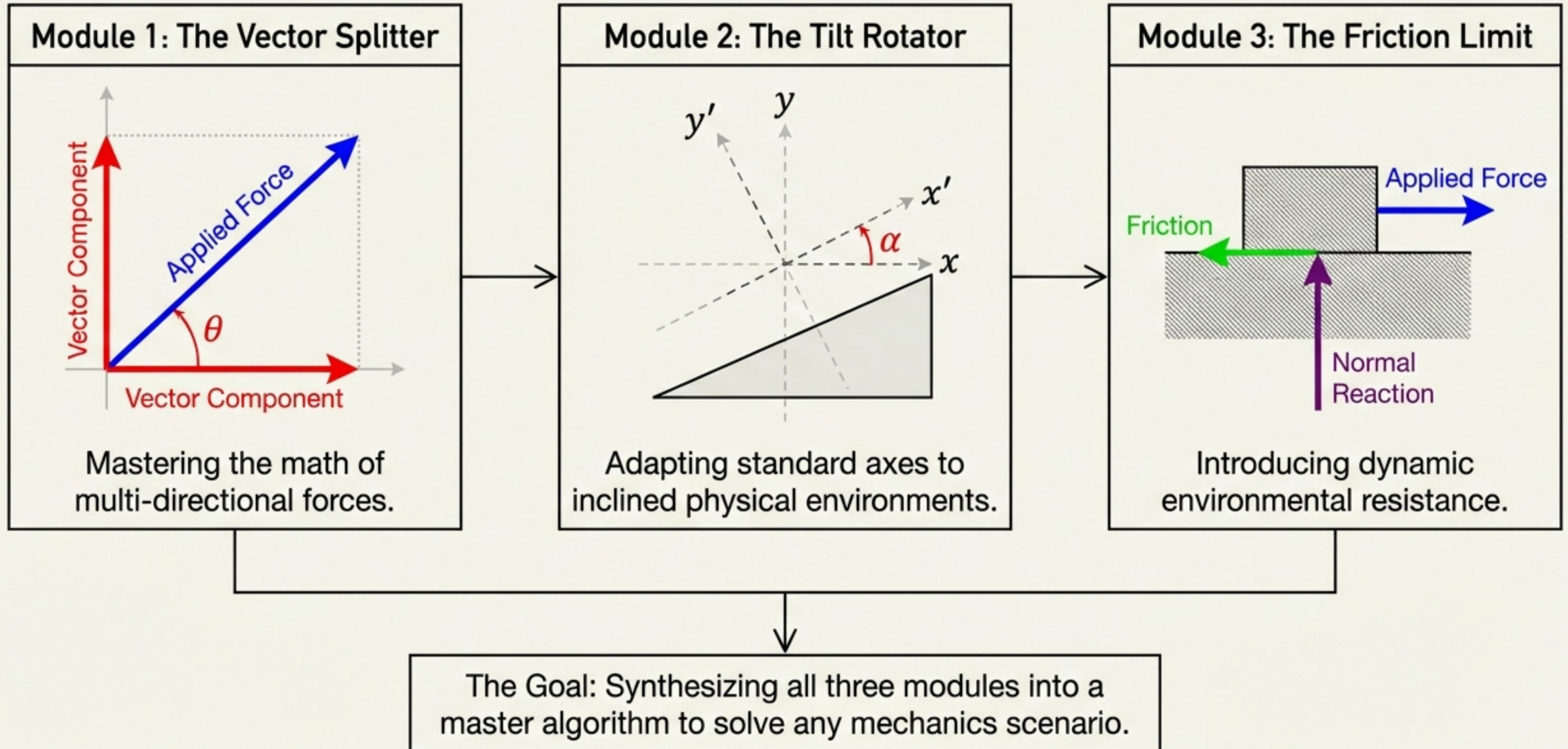
MECHANICS SCHEMATIC: FORCES & FRICTION

A visual diagnostic manual for A-Level Mathematics (Chapter 5)

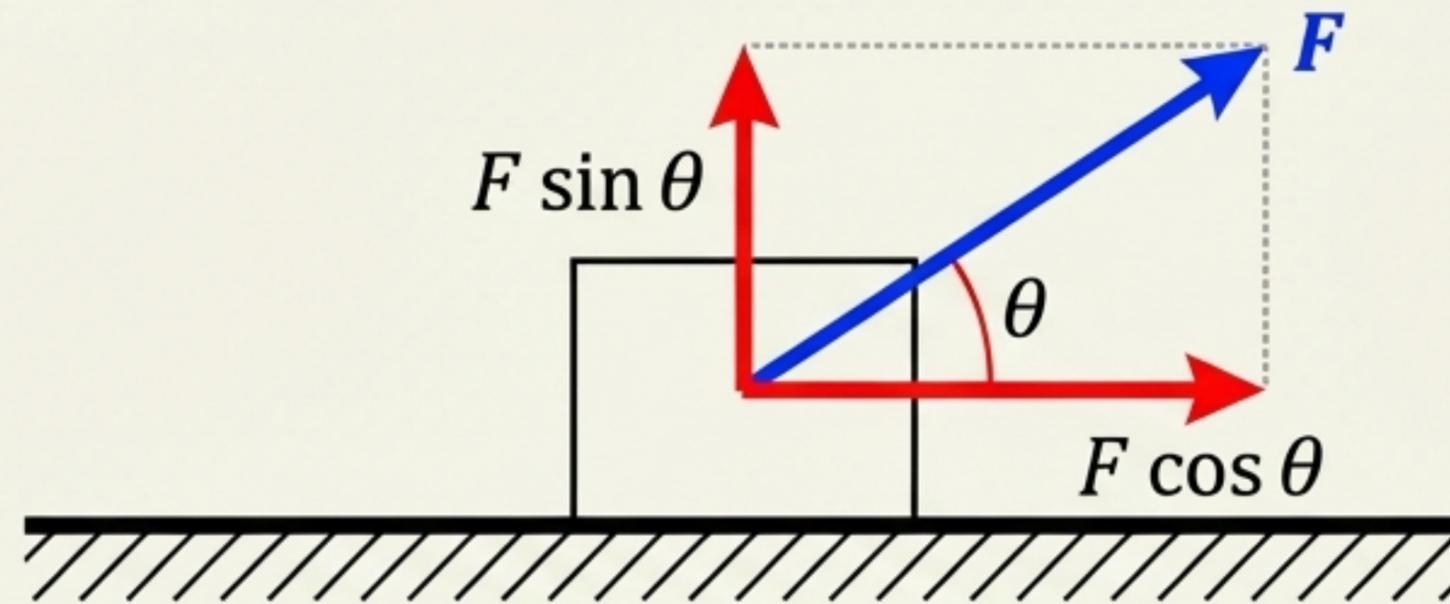


Schematic Legend	
	Applied Force (F, T)
	Components / Weight (mg)
	Friction (F_r)
	Normal Reaction (R)

System Architecture: The Learning Path

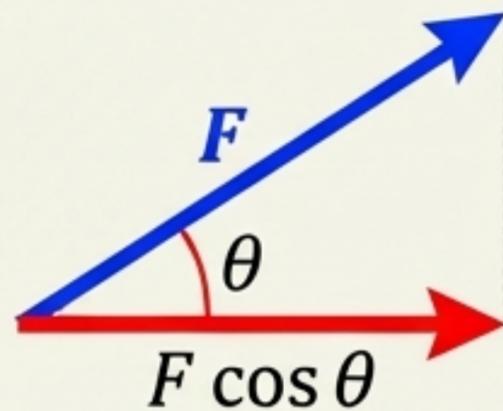


Module 1: The Vector Splitter



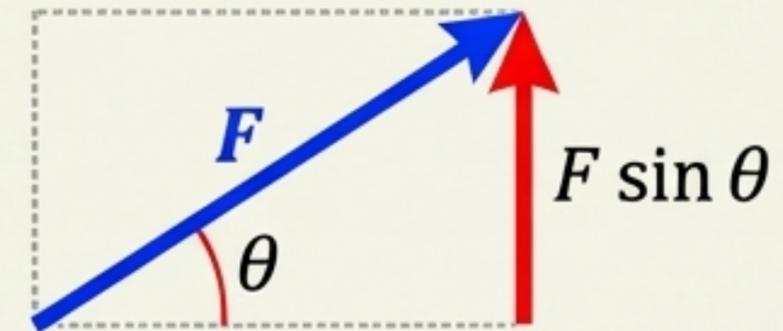
The Shadow Metaphor

Horizontal Component



The forward drive.

Vertical Component

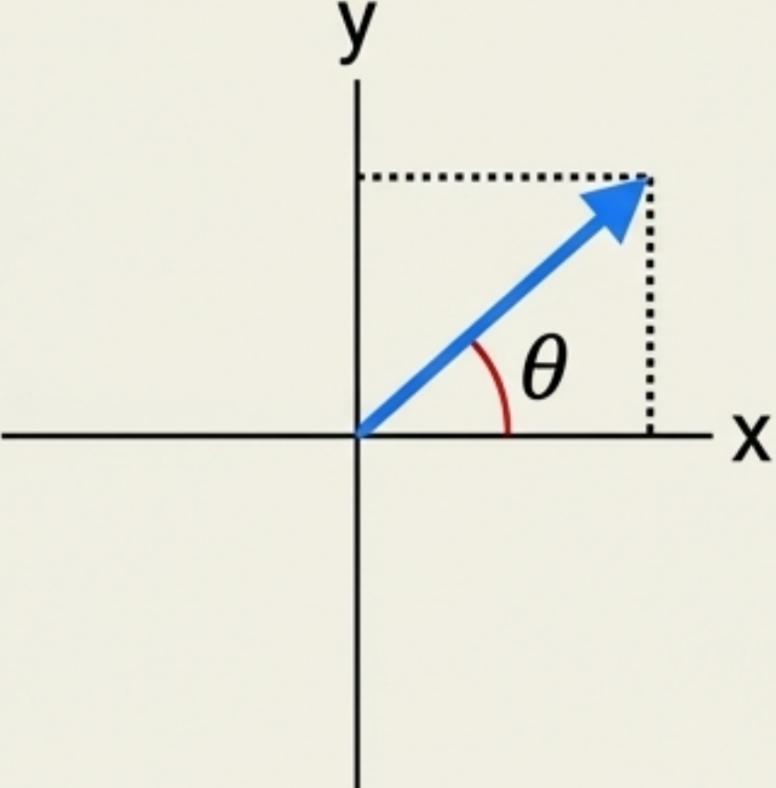
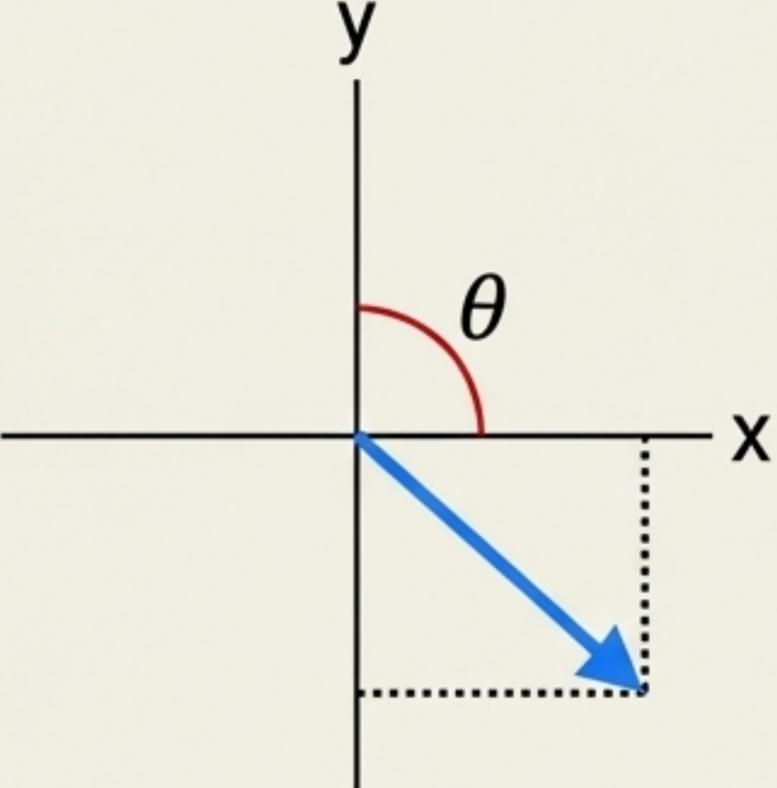


The lifting effect.

Any diagonal force acts in two directions simultaneously. We resolve the force to find its exact mechanical impact along a specific axis.

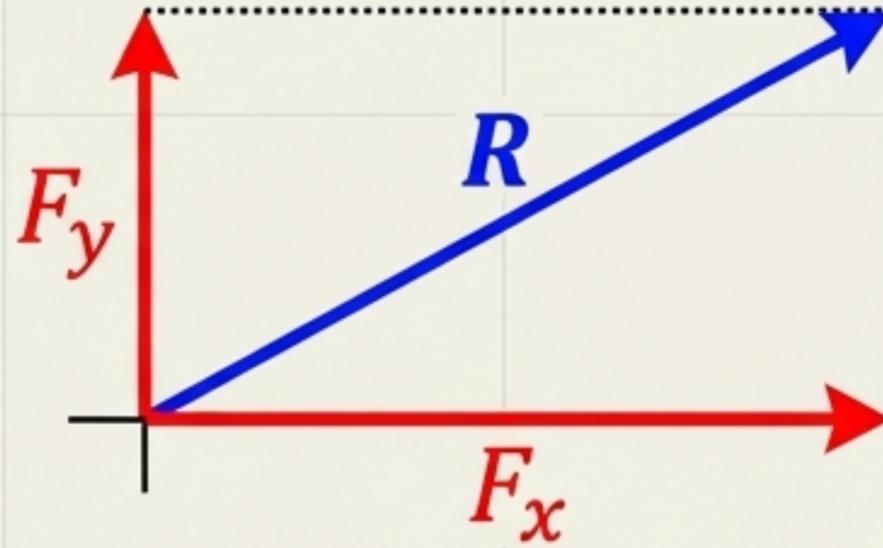
The Resolution Diagnostic Matrix

The Golden Rule: Geometry dictates the function, not the axis.

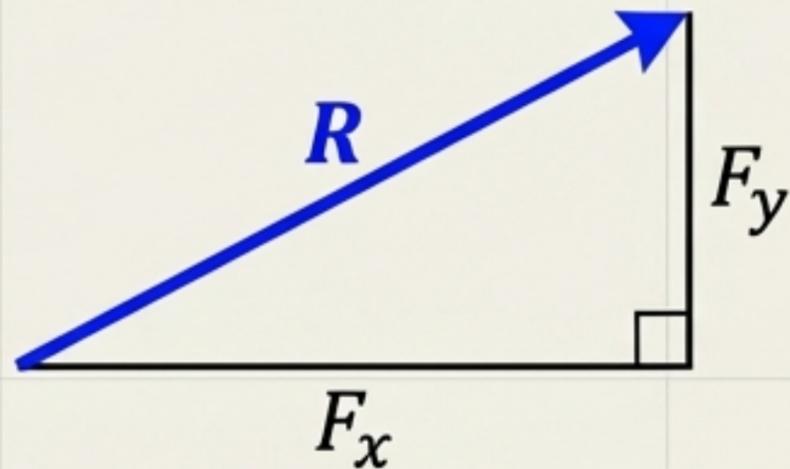
Angle to the Horizontal	Angle to the Vertical
 <p data-bbox="426 1339 1382 1534"><i>x</i>-component = $F \cos \theta$ <i>y</i>-component = $F \sin \theta$</p>	 <p data-bbox="1942 1339 2898 1534"><i>x</i>-component = $F \sin \theta$ <i>y</i>-component = $F \cos \theta$</p>

Cos resolves INTO the angle (squashing the angle).
Sin resolves AWAY from the angle (opening the angle).

Reverse Engineering: Finding the Resultant



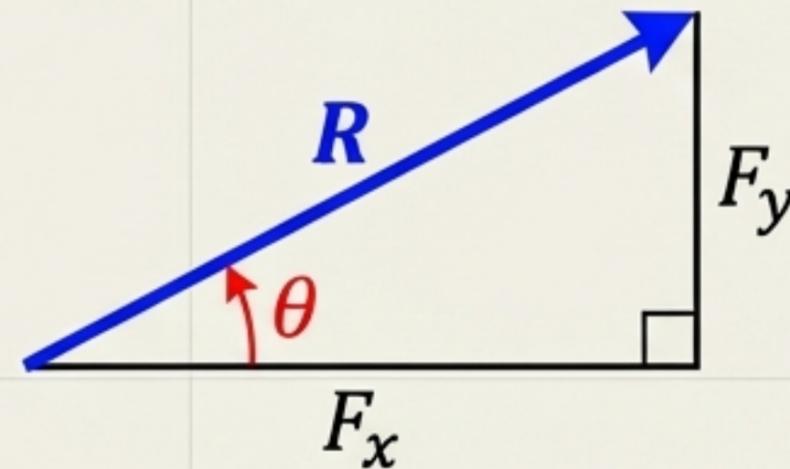
Step 1: Magnitude (The Engine)



$$|R| = \sqrt{F_x^2 + F_y^2}$$

(Pythagoras' Theorem)

Step 2: Direction/Bearing (The Steering)



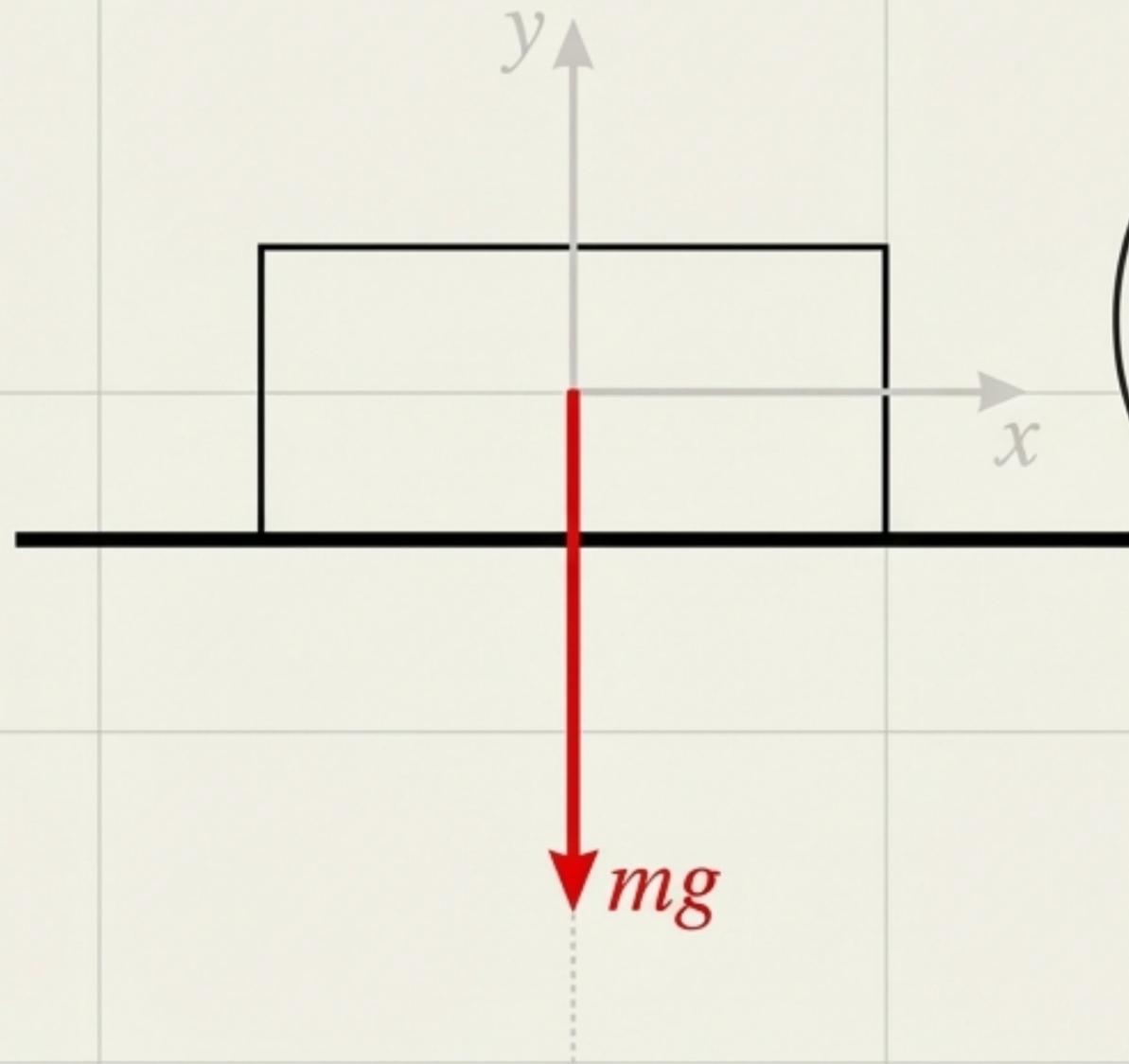
$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{F_y}{F_x}$$

Consultant's Whisper

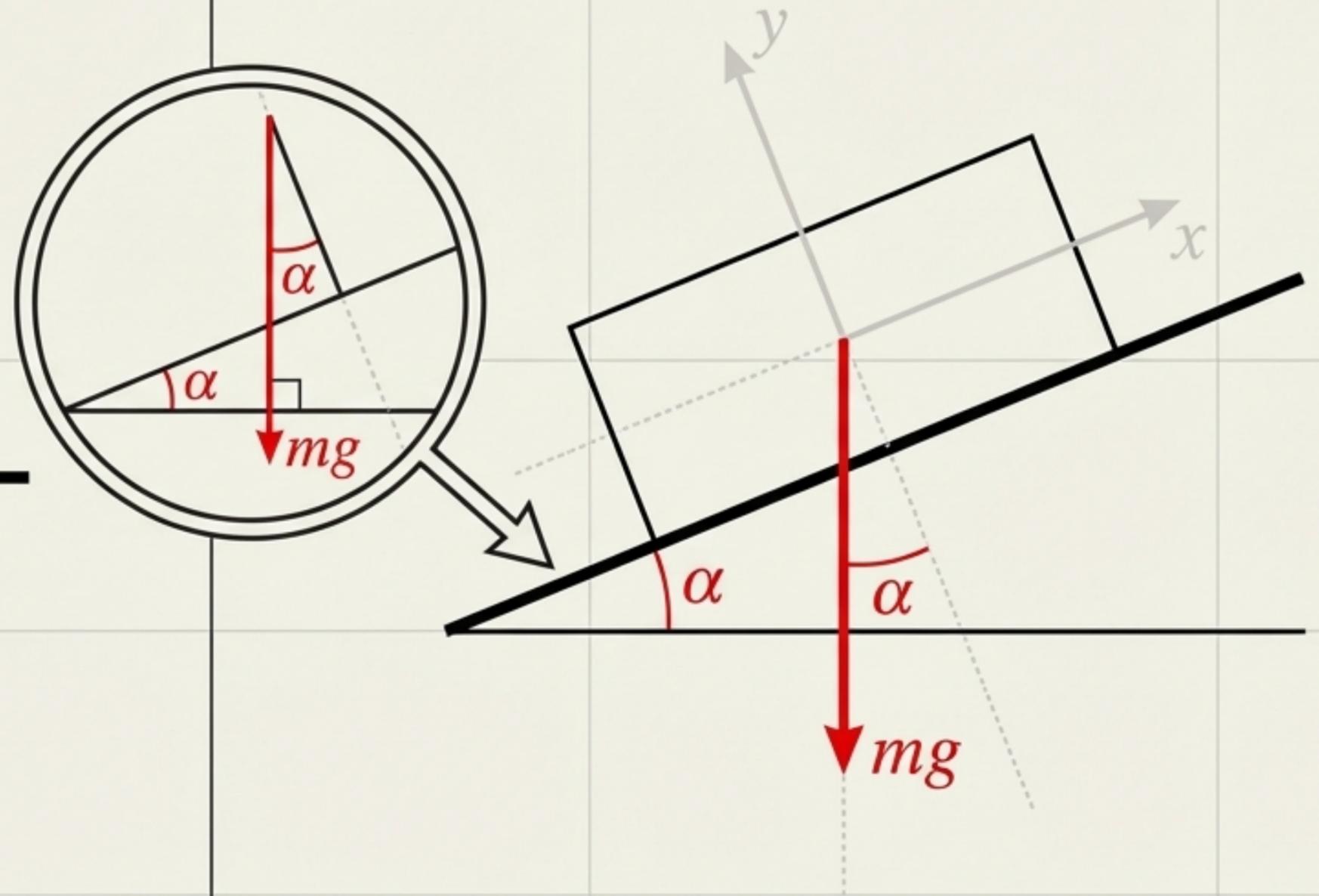
Always draw the physical triangle. It prevents sign errors and ensures ensures your calculated bearing maps to the real world.

Module 2: The Tilt Rotator

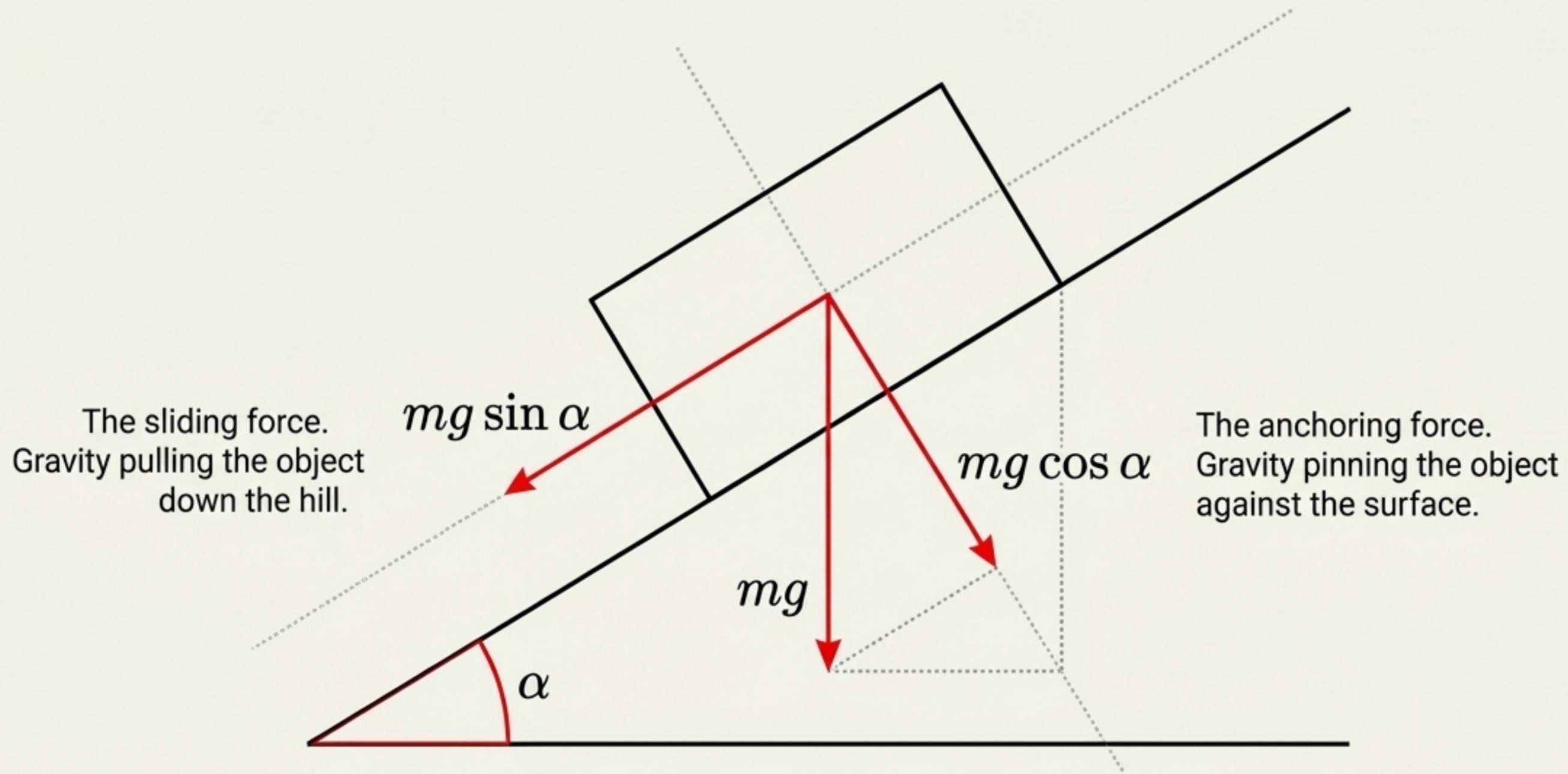
State 1: Horizontal



State 2: The Tilt



Decoding Gravity on a Slope

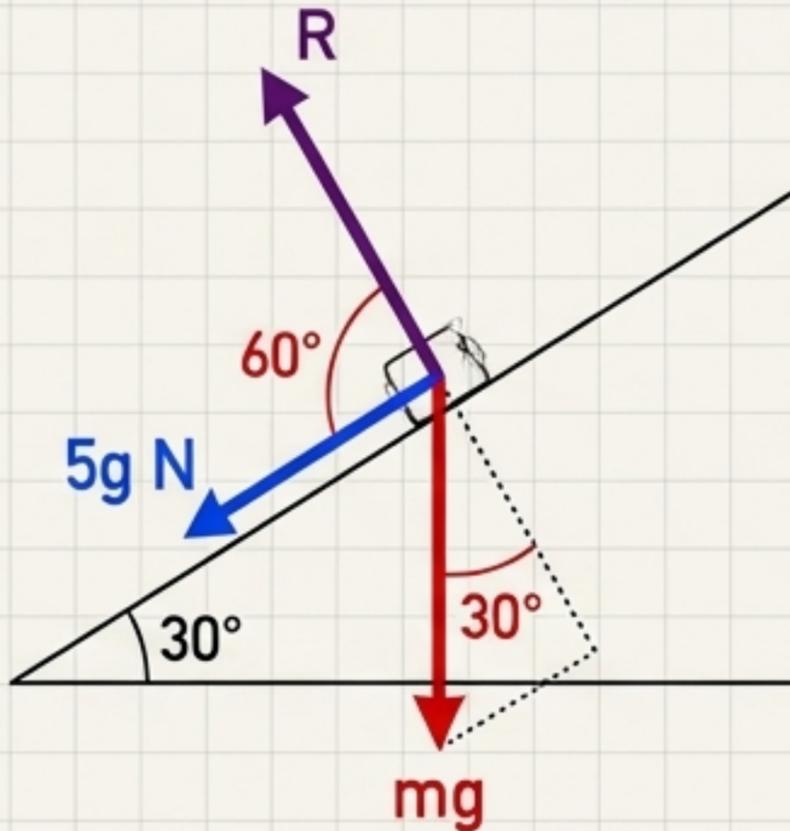


$R(\nearrow) \Rightarrow$ Normal Reaction $R = mg \cos \alpha$ (Assuming no other vertical forces)

Execution Matrix: Pushing Up a Slope

Context: A particle of mass m is pushed up a 30° smooth slope by a $5g$ N force acting at 60° to the plane, accelerating at 0.5 ms^{-2} . Find m .

[1] The Visual State



[2] The Math Engine

$$R(\searrow): R - 5g \sin 30^\circ - mg \cos 30^\circ = 0$$

$$R(\nearrow): 5g \cos 30^\circ - mg \sin 30^\circ = m(0.5)$$

$$m = 4.54 \text{ kg}$$

[3] The Consultant's Whisper

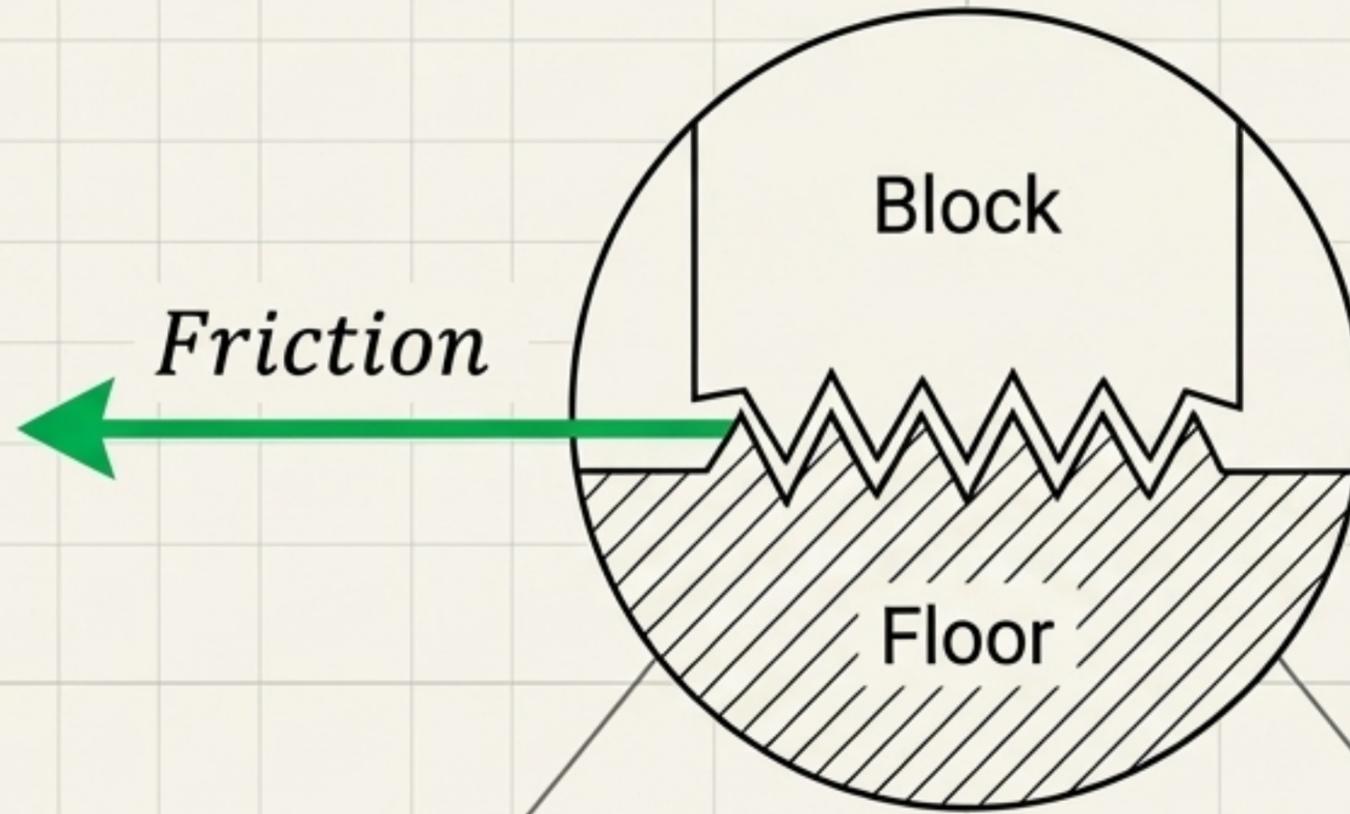
Hint

Resolve parallel first. Since we only need m and know acceleration, $F=ma$ parallel to the slope gives the answer without needing R .

Watch out

The applied force is pushing *into* the plane at 60° . Its component is $5g \cos 60^\circ$ up the plane, but $5g \sin 60^\circ$ *into* the plane.

Module 3: The Friction Limit



Friction is a reactive force that opposes motion between rough surfaces.

μ (**Coefficient of Friction**):
A measure of the roughness between two specific materials.
(Smooth surface $\Rightarrow \mu = 0$).

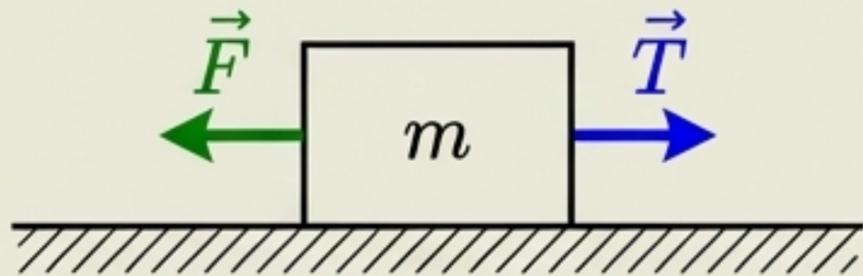
$$F_{MAX} = \mu R$$

R (**Normal Reaction**):
How hard the surfaces are physically being pressed together.

Core Insight: Friction isn't constant. It scales based on how heavily the object is anchored to the slope (R).

The Friction Spectrum

State 1: Stationary

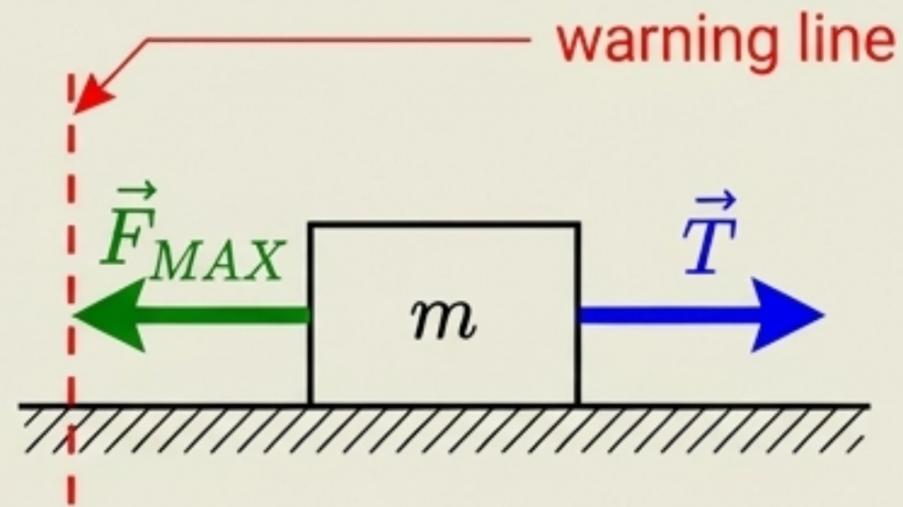


Condition: Tendency to move, but locked.

$$F = T. \quad F < \mu R$$

(Friction simply matches the applied force).

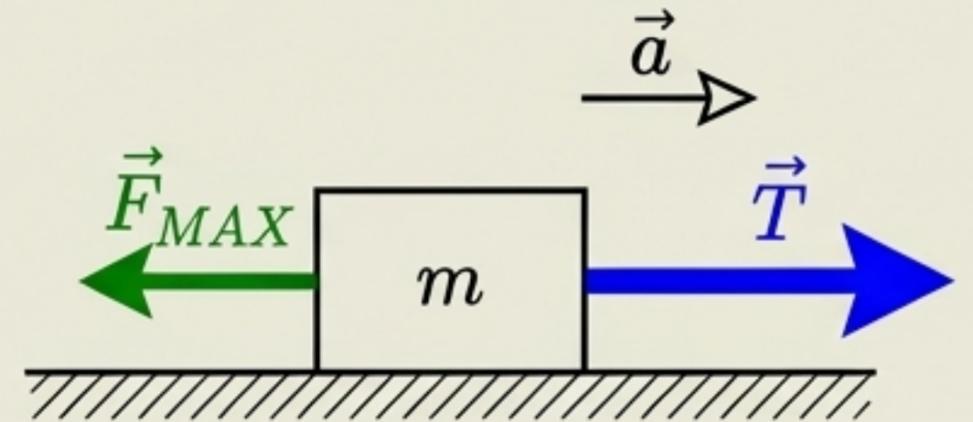
State 2: Limiting Equilibrium



Condition: On the absolute point of moving.

$$F = F_{MAX} = \mu R$$

State 3: Accelerating



Condition: Force defeats friction.

$$F = \mu R \quad \text{and} \quad T - F = ma$$

The Friction Thermometer



1 Pushing lightly

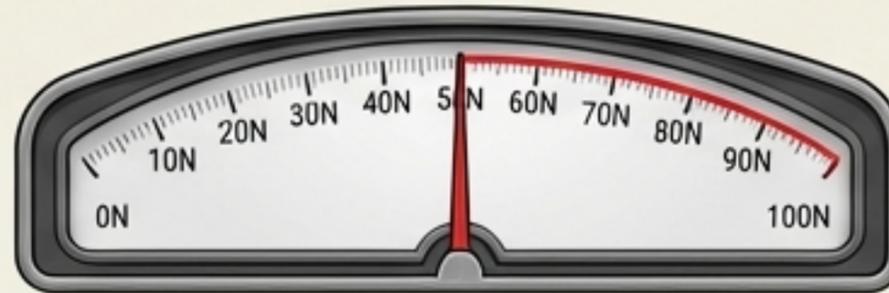
Applied force is 10N.
Friction gives exactly 10N back.



2 Pushing harder

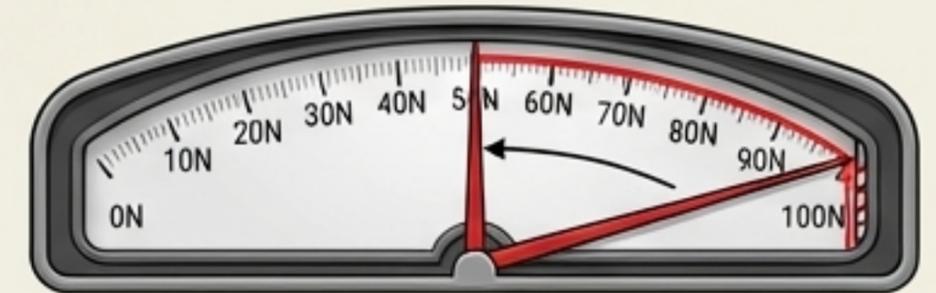
Applied force rises to 49N.
The system is in Limiting Equilibrium.

$$F_{MAX} = \mu R$$



3 Breaking the limit

Applied force goes to 100N.
Friction snaps off at 49N.
The remaining 51N becomes pure acceleration ($F=ma$).



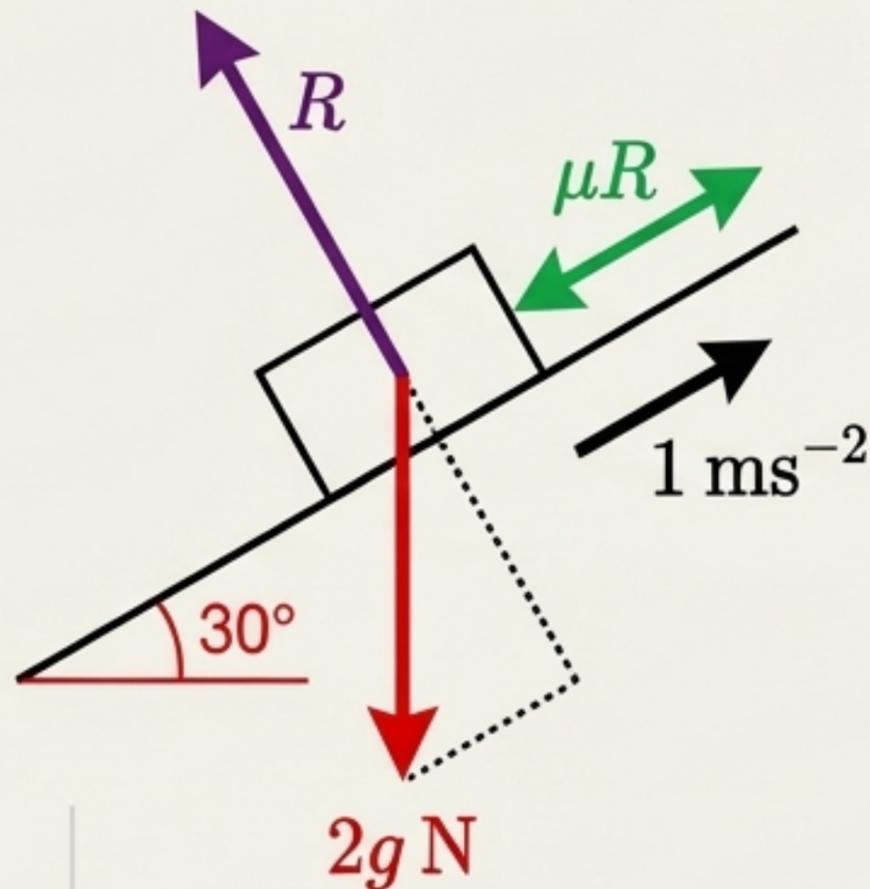
$$\begin{aligned} & 100\text{N (Applied)} - 49\text{N (Friction)} \\ & = 51\text{N (Acceleration Force)} \end{aligned}$$

Key Takeaway: Friction is lazy. It only uses as much force as required to stop motion, up to its absolute breaking point (μR).

Execution Matrix: The Rough Incline

Context: Example 10. A 2kg mass slides down a 30° rough slope, accelerating at 1 ms⁻². Find μ .

[1] The Visual State



[2] The Math Engine

$$R(\perp): R = 2g \cos 30^\circ = 16.974 \text{ N}$$

$$R(\parallel): 2g \sin 30^\circ - \mu R = 2(1)$$

Substitute R:

$$9.8 - \mu(16.974) = 2$$

$$\Rightarrow \mu = 0.460$$

[3] The Consultant's Whisper

Friction Direction

The particle is sliding down, so friction must point UP the slope.

The Substitution Loop

Rough slope problems are always a two-step loop.

Resolve perpendicular to find R.

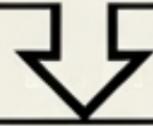
Plug R into $F = \mu R$.

Use that substituted value in your parallel $F=ma$ equation.

The Master Protocol

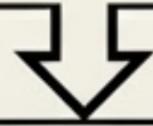
Step 1: The Blueprint (FBD)

Draw the particle. Map every force (Applied, Weight, Reaction, Friction). Add the acceleration arrow off to the side.



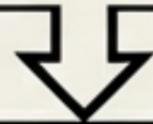
Step 2: Align the Grid (Axes)

Choose your x/y axes. If on a slope, physically rotate axes to be parallel and perpendicular to the plane.



Step 3: The Vector Split

Resolve all diagonal forces into components along your newly chosen axes using **Cosine** (into the angle) and **Sine** (away from the angle).



Step 4: The Physics Engine (Equations)

- **Perpendicular to motion:** Sum of forces = 0 (Find R)
- **Parallel to motion:** Net Force = ma
- **If rough:** Link the two equations with $F \leq \mu R$

The Engineer's Final Checklist

1. Resolution

$F \cos \theta$ (squashing into the angle)

$F \sin \theta$ (opening away from the angle)

2. Gravity on Slopes

Down the slope = $mg \sin \alpha$

Into the slope = $mg \cos \alpha$

3. Friction States

Stationary: $F < \mu R$

Point of slipping: $F = \mu R$

Moving: $F = \mu R$

4. Equilibrium vs Dynamics

Equilibrium $\Rightarrow a = 0$ (Forces balance perfectly)

Dynamics $\Rightarrow a > 0$ (Resultant force drives $F=ma$)

Trust the schematic. Draw the diagram. The geometry holds the answer.