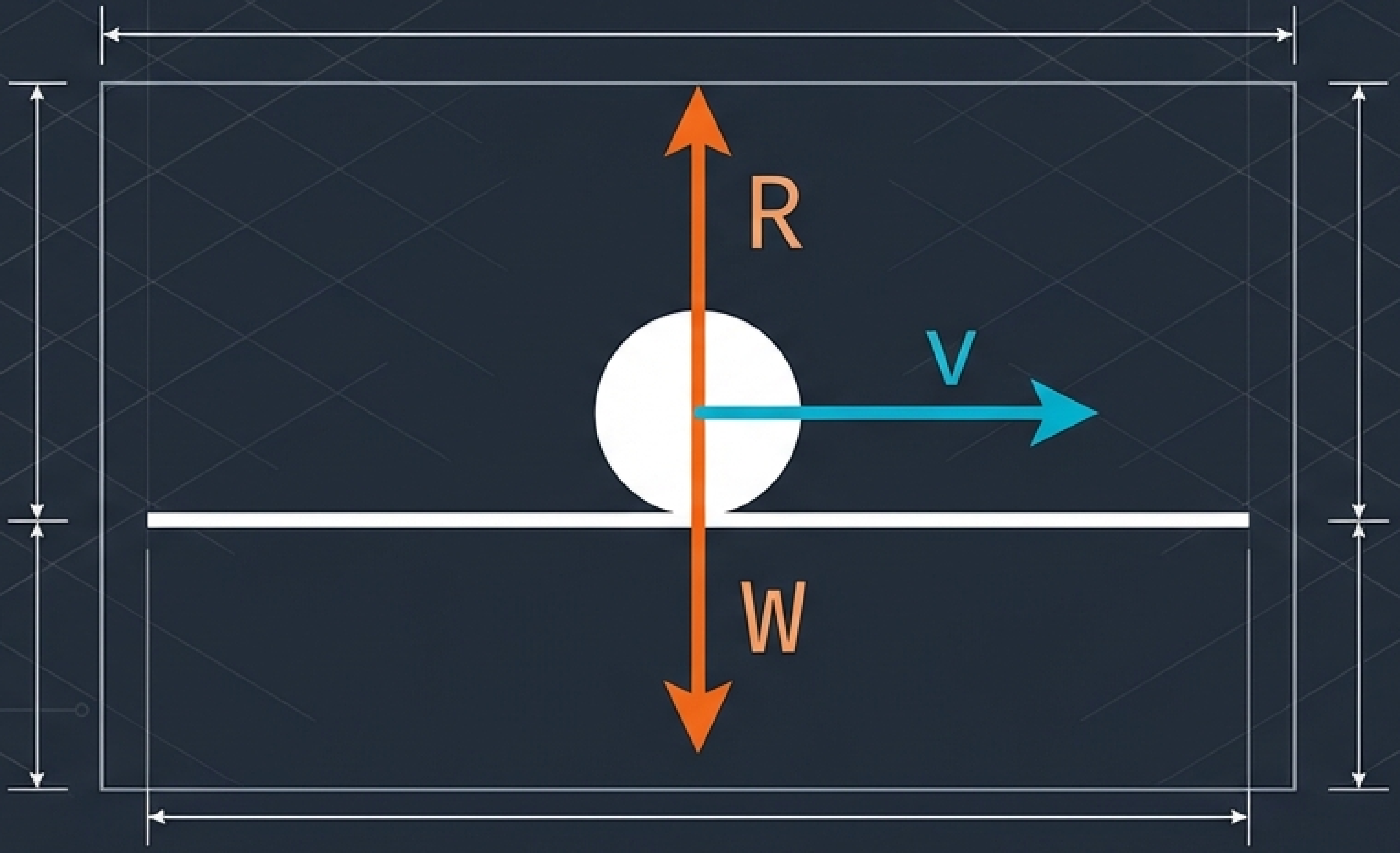
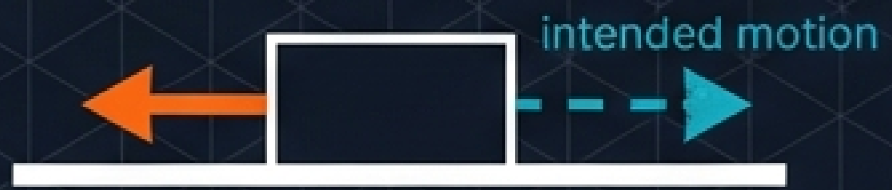
Dynamics of a Particle

The Blueprint for Straight-Line Motion



Chapter 4 Masterclass: Diagrams, Vectors, and Complex Systems.

The Forces Inventory

Force Type	Symbol	Visual Rule (Direction)	Magnitude Formula
Weight	W		$W = mg$ $g = 9.8 \text{ m s}^{-2}$
Normal Reaction	R		Variable (Depends on equilibrium)
Friction	F or $F_{\max}$		Variable (Opposes motion)
Tension / Thrust	T		Variable

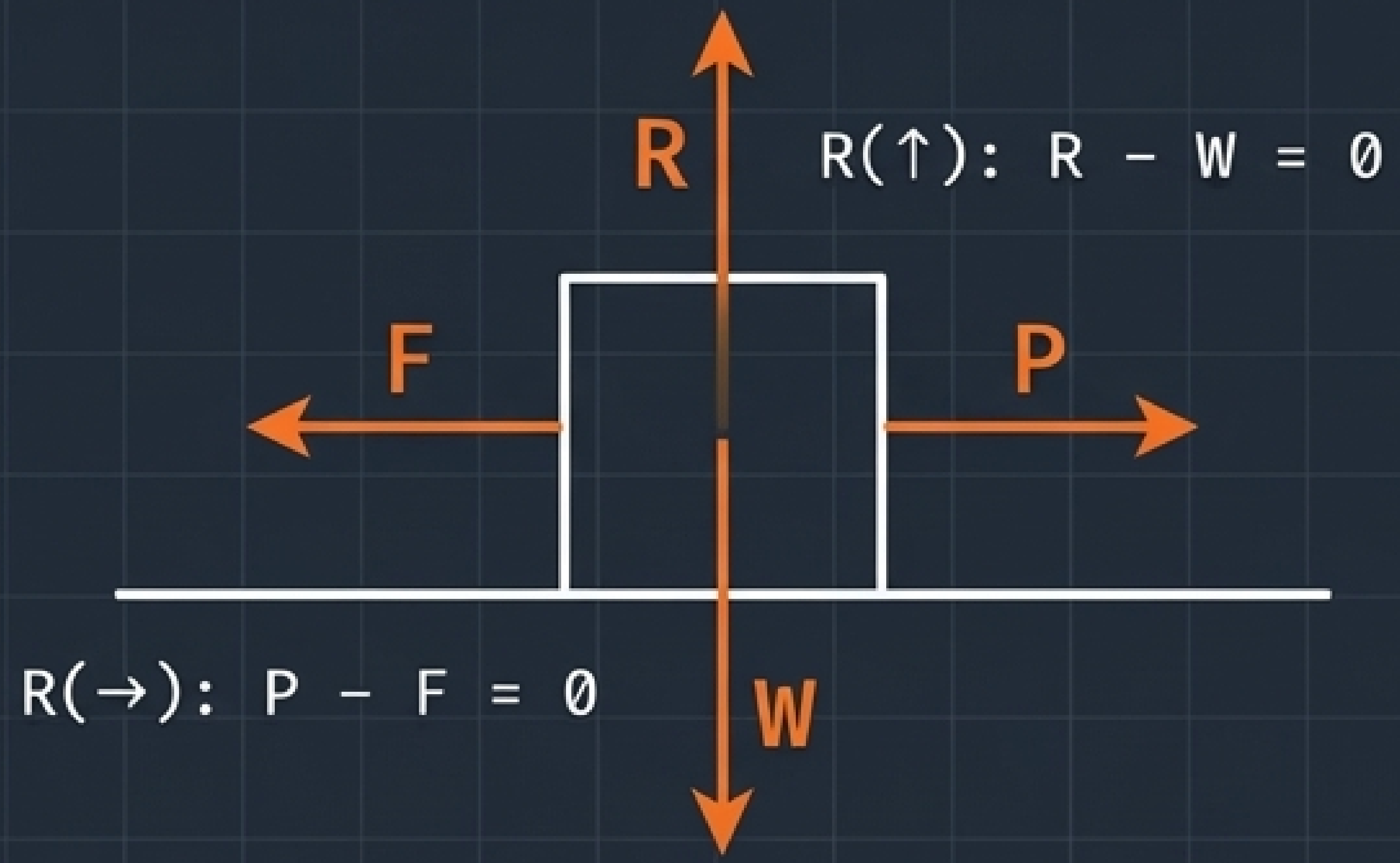
Translating Reality to Diagrams

Newton's First Law

An object at rest or moving with constant velocity has no unbalanced forces.

The Rule: This is called Equilibrium. The resultant force is zero (R = 0).

The Equilibrium Balance Scale



Watch Out: Constant velocity means acceleration is zero. Treat it exactly like an object at rest.

Quantifying Forces: The Vector Language

Forces can be written using **i** (east/horizontal) and **j** (north/vertical) unit vectors, or as column vectors.

Math Block

$$(2\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) + (-3\mathbf{i} + 2\mathbf{j}) = \mathbf{R}$$

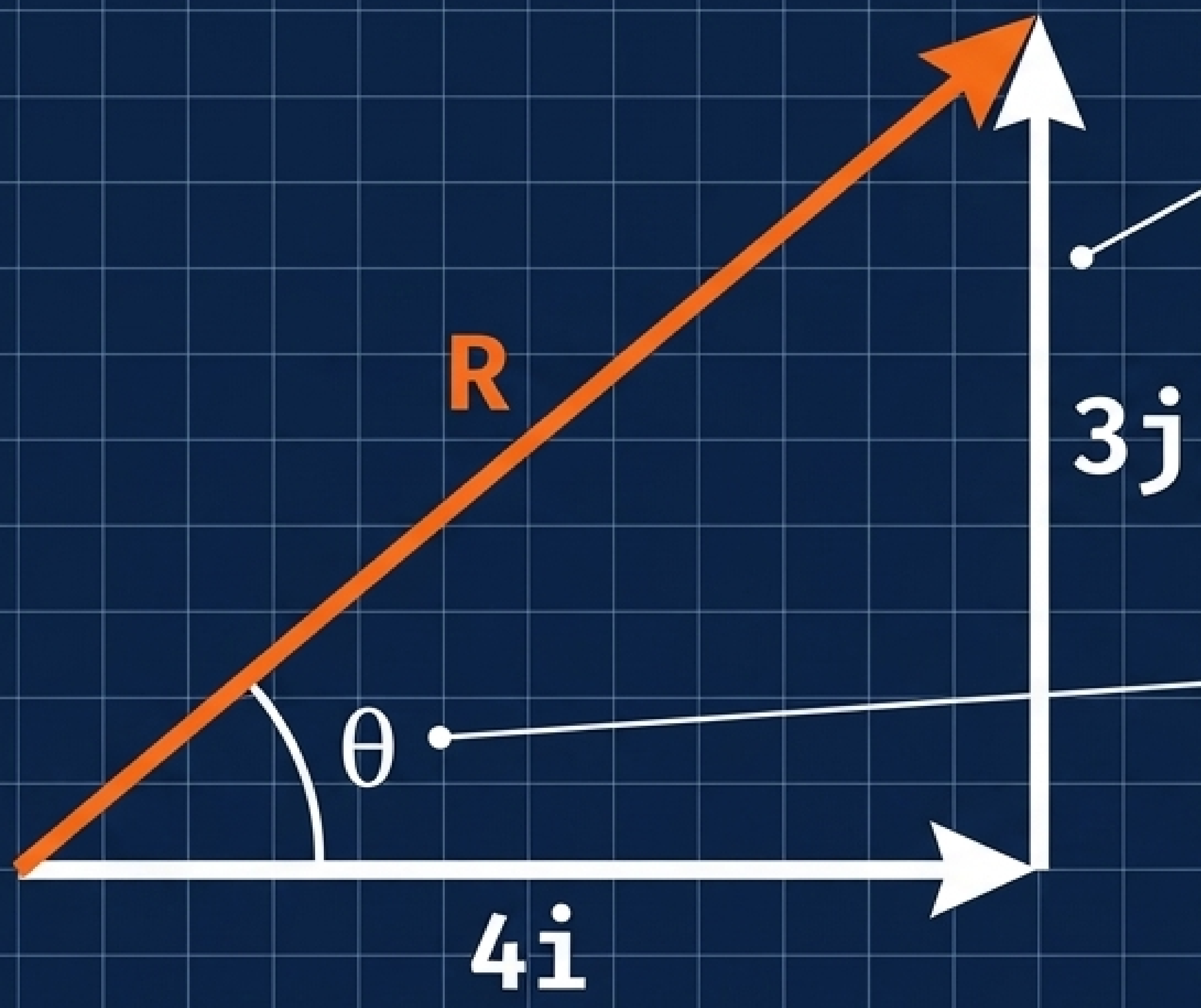
$$(2 + 4 - 3)\mathbf{i} = 3\mathbf{i}$$

$$(3 - 1 + 2)\mathbf{j} = 4\mathbf{j}$$

$$\text{Resultant} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \text{N}$$

When a particle is in equilibrium, the resultant vector is $0\mathbf{i} + 0\mathbf{j}$.

Resultant Forces: Magnitude & Bearing



Magnitude (|R|)

$$|\mathbf{R}| = \sqrt{x^2 + y^2}$$

$$\sqrt{4^2 + 3^2} = 5 \text{ N}$$

Direction (Bearing)

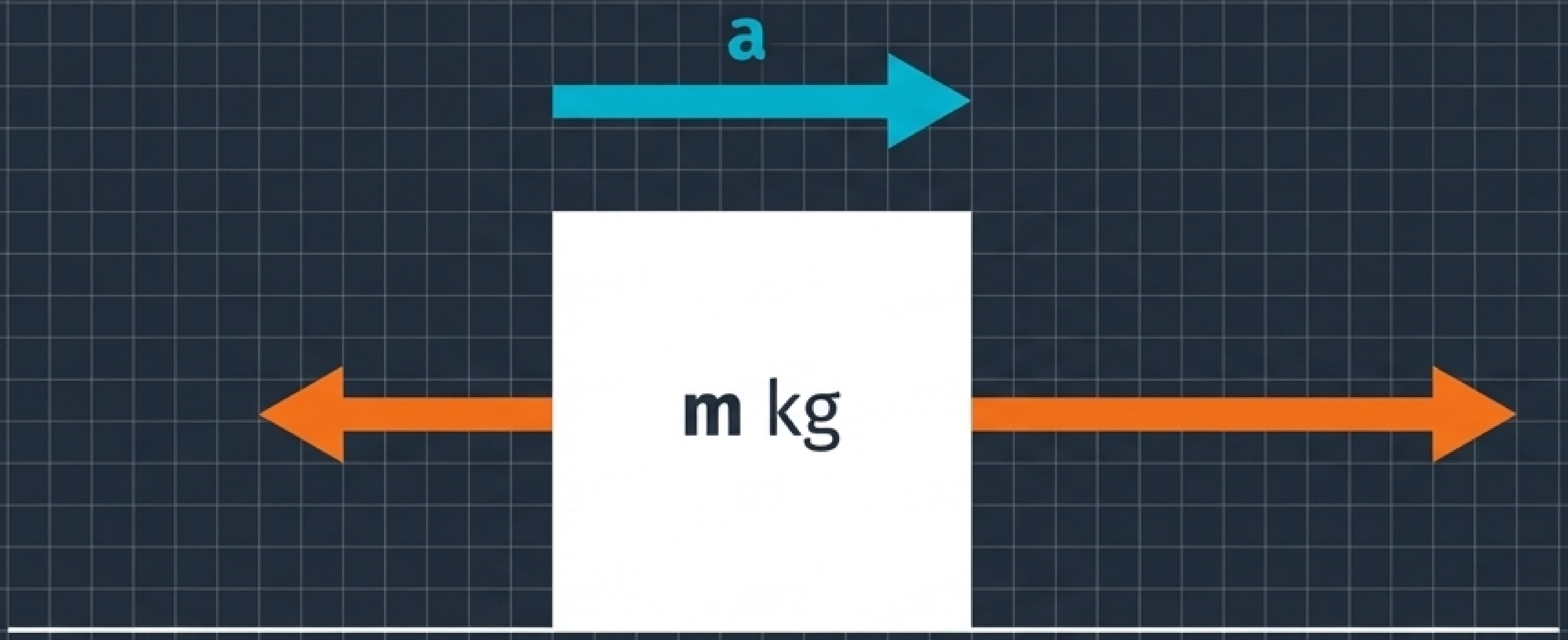
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

$$\theta = 36.9^\circ$$

Bearings are measured clockwise from North: 90° - 36.9° = 053.1°

The Core Engine: F = ma

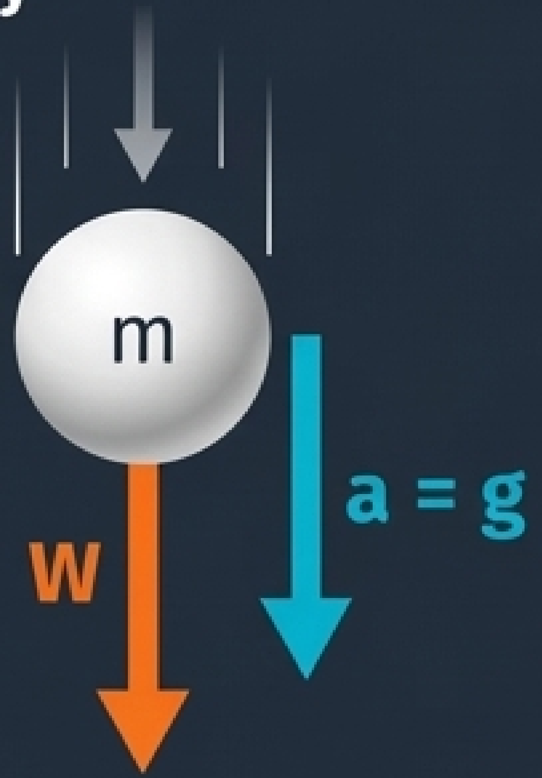
Newton's Second Law: A non-zero resultant force causes a particle to accelerate in the direction of that resultant force.



Resultant Force (F) = Mass (m) × Acceleration (a)

Sub-Concept

Gravity



Gravity is a specific application of F = ma.

Weight (W) = mg

(Always acts vertically downwards, $g = 9.8\ \text{m s}^{-2}$)

Motion in Two Dimensions

Vector F = ma

Acceleration can be written as a 2D vector in the form (pi + qj) m s^{-2}.

$$\mathbf{F} = m\mathbf{a}$$

- If mass is 0.5 kg and $\mathbf{F} = (3\mathbf{i} + 8\mathbf{j})$ N:

$$(3\mathbf{i} + 8\mathbf{j}) = 0.5 \times \mathbf{a}$$

$$\mathbf{a} = (6\mathbf{i} + 16\mathbf{j}) \text{ m s}^{-2}$$

Vector Kinematics

Constant acceleration formulae still apply, just replace scalars with vectors.

$$v = u + at \quad \rightarrow \quad \mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$s = ut + \frac{1}{2}at^2 \quad \rightarrow \quad \mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

Keep **i** and **j** separated throughout your entire calculation.

Worked Example: 2D Motion

Two forces, $\mathbf{F}_1 = (2\mathbf{i} + 7\mathbf{j})$ N and $\mathbf{F}_2 = (-3\mathbf{i} + \mathbf{j})$ N, act on a particle of mass 0.25 kg. Find acceleration.

Step 1: Find Resultant Force

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$$

$$\mathbf{R} = (2\mathbf{i} + 7\mathbf{j}) + (-3\mathbf{i} + \mathbf{j}) = (-\mathbf{i} + 8\mathbf{j})\ \text{N}$$

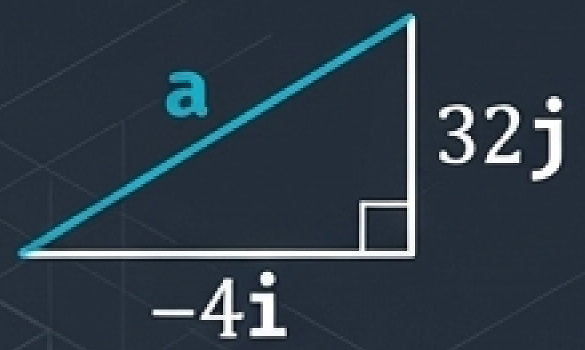
Step 2: Apply Engine (F=ma)

$$(-\mathbf{i} + 8\mathbf{j}) = 0.25\mathbf{a}$$

Multiply by 4 to isolate a

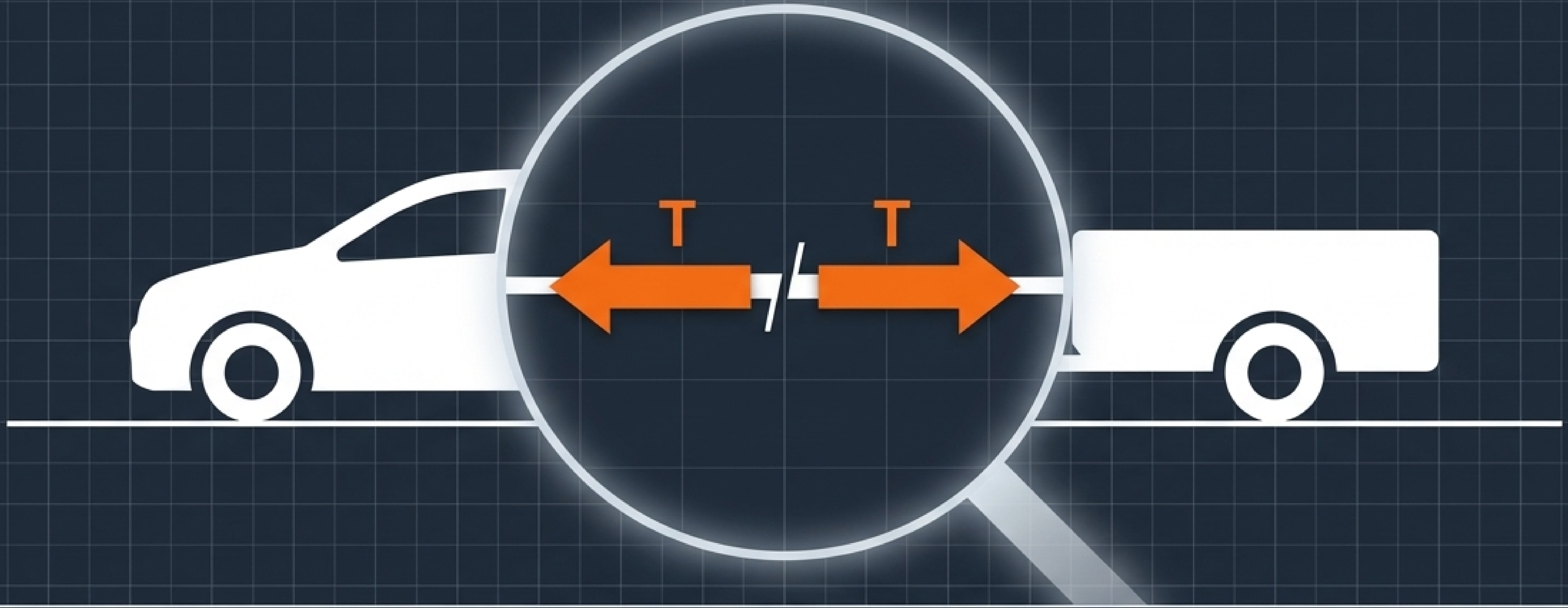
$$\mathbf{a} = (-4\mathbf{i} + 32\mathbf{j})\ \text{m s}^{-2}$$

Step 3: Magnitude (Optional target)



$$|\mathbf{a}| = \sqrt{(-4)^2 + 32^2} = \sqrt{16 + 1024} = \sqrt{1040}\ \text{m s}^{-2}$$

Connected Particles & Newton's 3rd Law

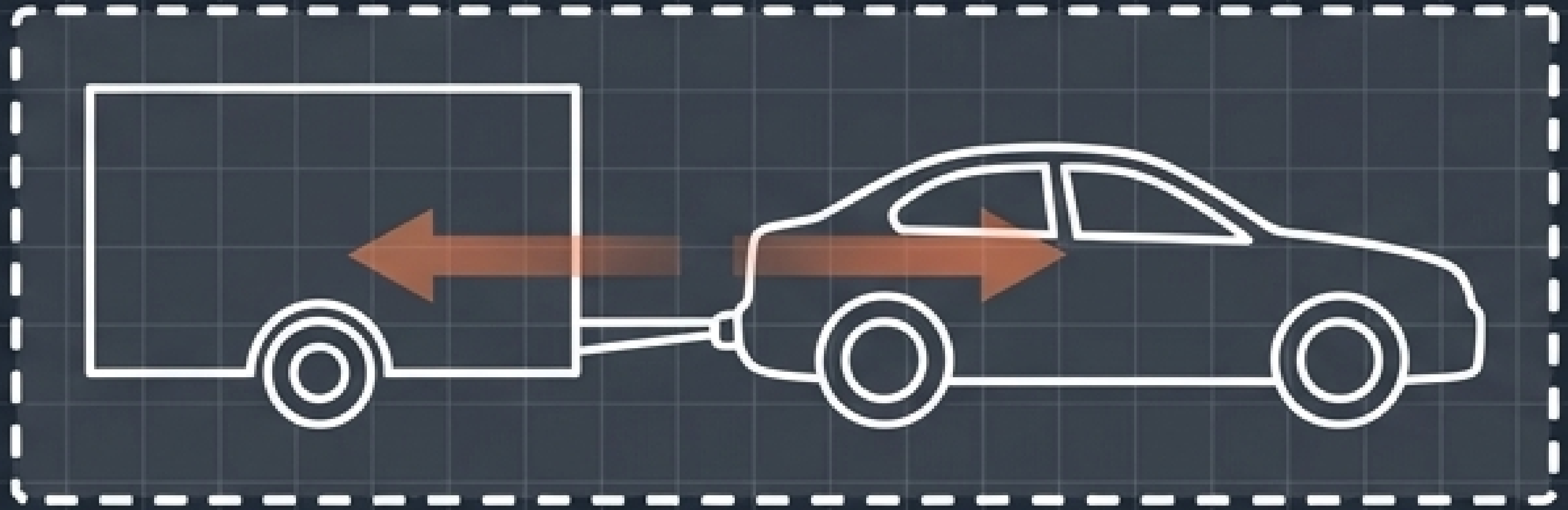


Concept Block: Newton's Third Law: Every action has an equal and opposite reaction.

Synthesis: When two bodies are connected, the force body A exerts on body B is equal in magnitude and opposite in direction to the force B exerts on A. These are internal forces. When viewing the system as a whole, they cancel out.

The Analytical Toggle: System vs. Particle Lens

The "Whole System" Lens

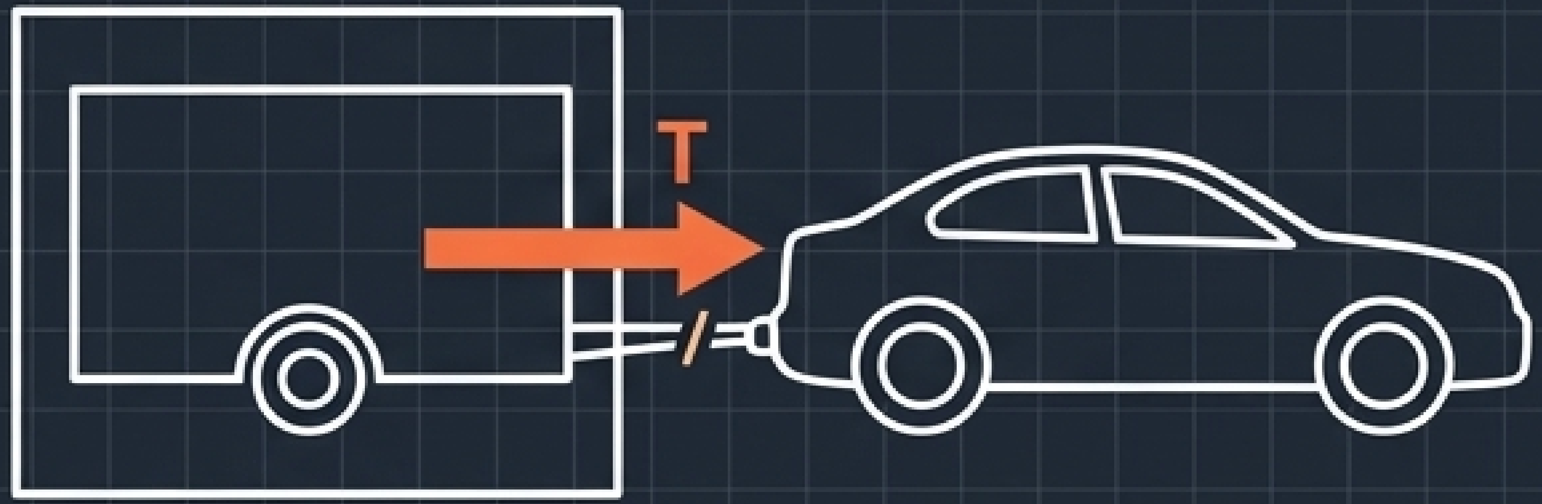


When to use: To find the global Acceleration (**a**) or total Driving Force.

$$F_{drive} - F_{friction(total)} = (m_1 + m_2)a$$

(Internal tension **T** vanishes)

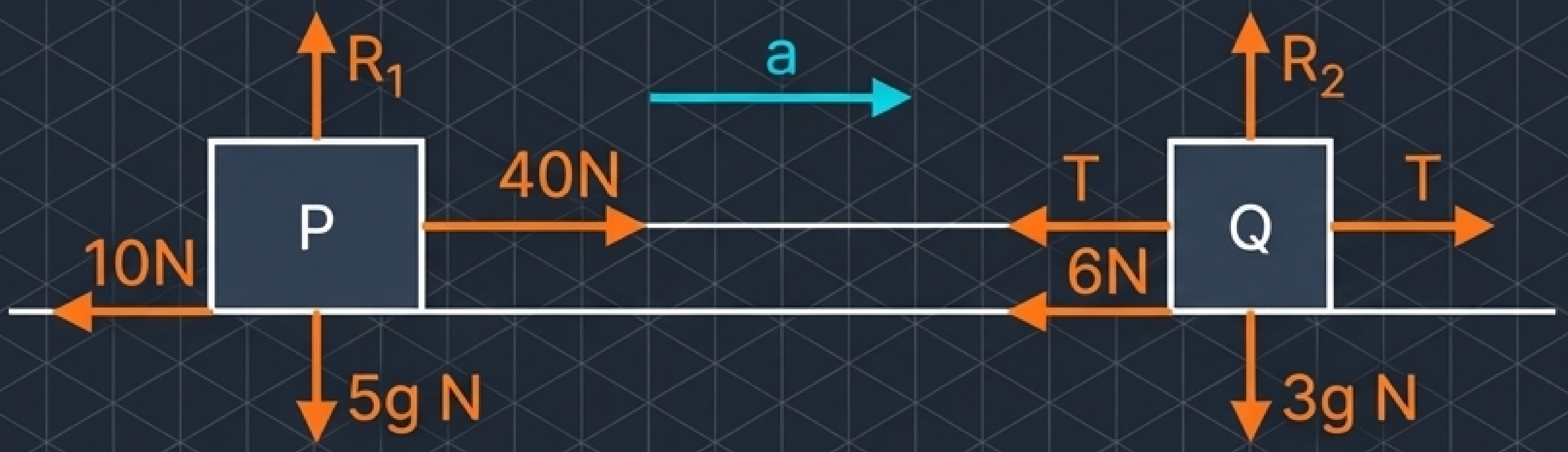
The "Individual Particle" Lens



When to use: To find internal forces like Tension (**T**) or Thrust.

$$T - F_{friction(trailer)} = (m_{trailer})a$$

Worked Example: The Car & Trailer



Step 1: Whole System

Find acceleration (a) by treating it as an 8kg block.

$R(\rightarrow): 40 - 10 - 6 = 8a$

$24 = 8a$

$a = 3 \text{ m s}^{-2}$

Step 2: Individual Particle

Find tension (T) by looking ONLY at particle Q (3kg).

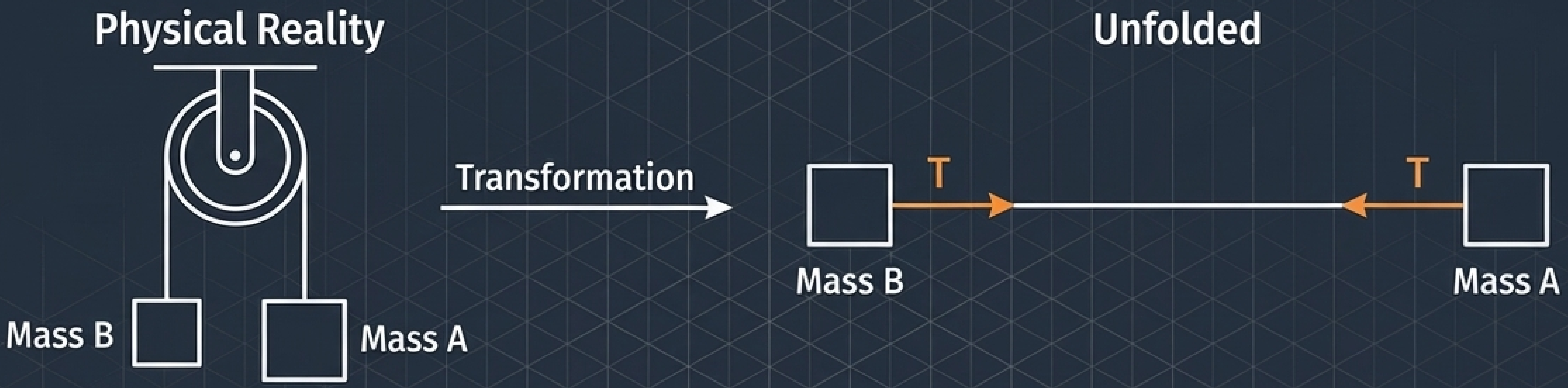
$R(\rightarrow): T - 6 = 3 \times a$

$T - 6 = 3(3)$

$T = 15 \text{ N}$

Pulleys: Bending the Axis of Motion

The Unfolded Concept



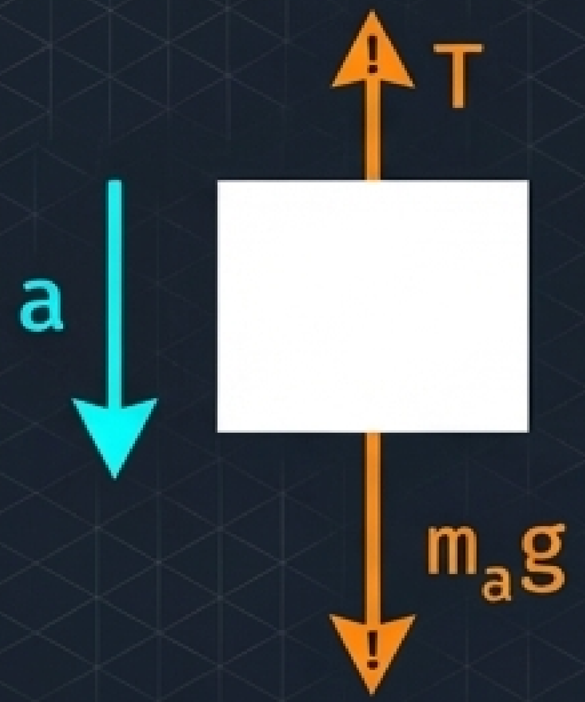
The Assumptions Dictionary

- Smooth Pulley → Tension (T) is exactly the same on both sides of the string.
- Inextensible String → Both masses share the exact same magnitude of Acceleration (a).

Key Takeaway: Because of these assumptions, we can link their equations of motion.

Pulley Equations of Motion

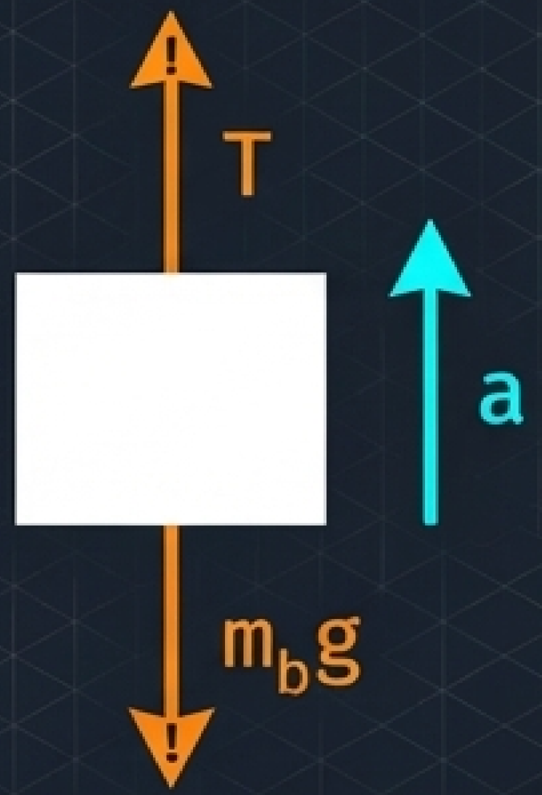
The Descending Mass (Particle A)



Resolve in the direction of motion (Downwards).

$$m_a g - T = m_a a \quad \text{(Eq 1)}$$

The Ascending Mass (Particle B)



Resolve in the direction of motion (Upwards).

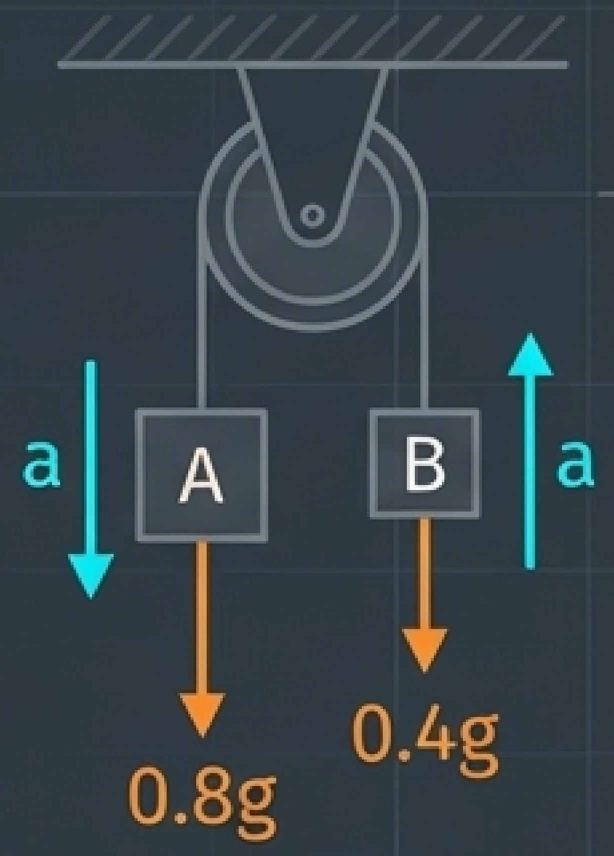
$$T - m_b g = m_b a \quad \text{(Eq 2)}$$

The Math Trick

Add Equation 1 and Equation 2 together. The +T and -T will always cancel out, leaving you an equation with just a!

Worked Example: The Smooth Pulley

Particle A (0.8 kg) and Particle B (0.4 kg) over a smooth pulley.



For A (Moving Down): $0.8g - T = 0.8a$

For B (Moving Up): $T - 0.4g = 0.4a$

The Merge (Add them together):

$$(0.8g - \cancel{T}) + (\cancel{T - 0.4g}) = 0.8a + 0.4a$$

$$0.4g = 1.2a$$

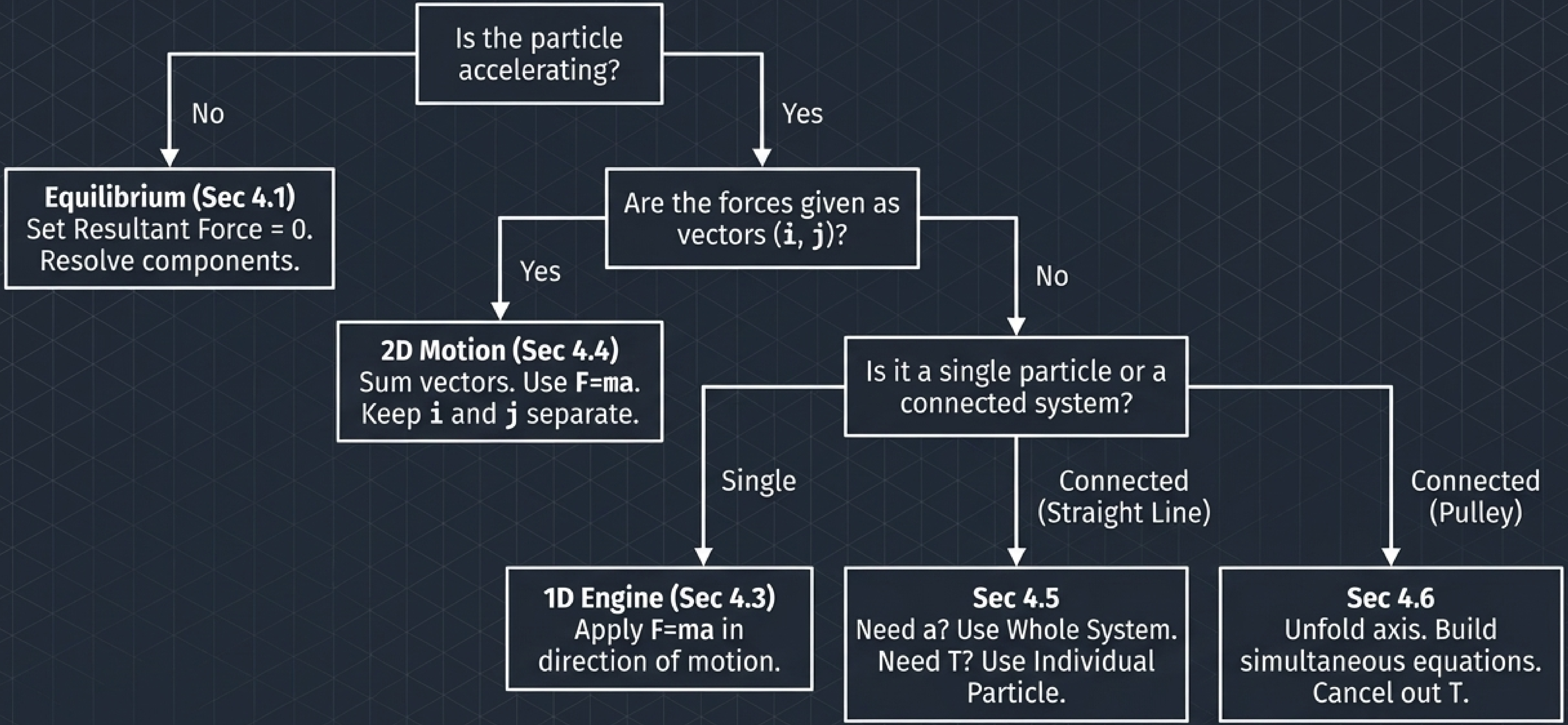
$$a = 0.4g / 1.2 = (1/3)g \text{ m s}^{-2}$$

Find Tension (T)

Substitute 'a' back into B's equation:

$$T = 0.4g + 0.4((1/3)g) = (8/15)g \text{ N}$$

Synthesis: The Master Algorithm



Follow the blueprint. Trust the math.