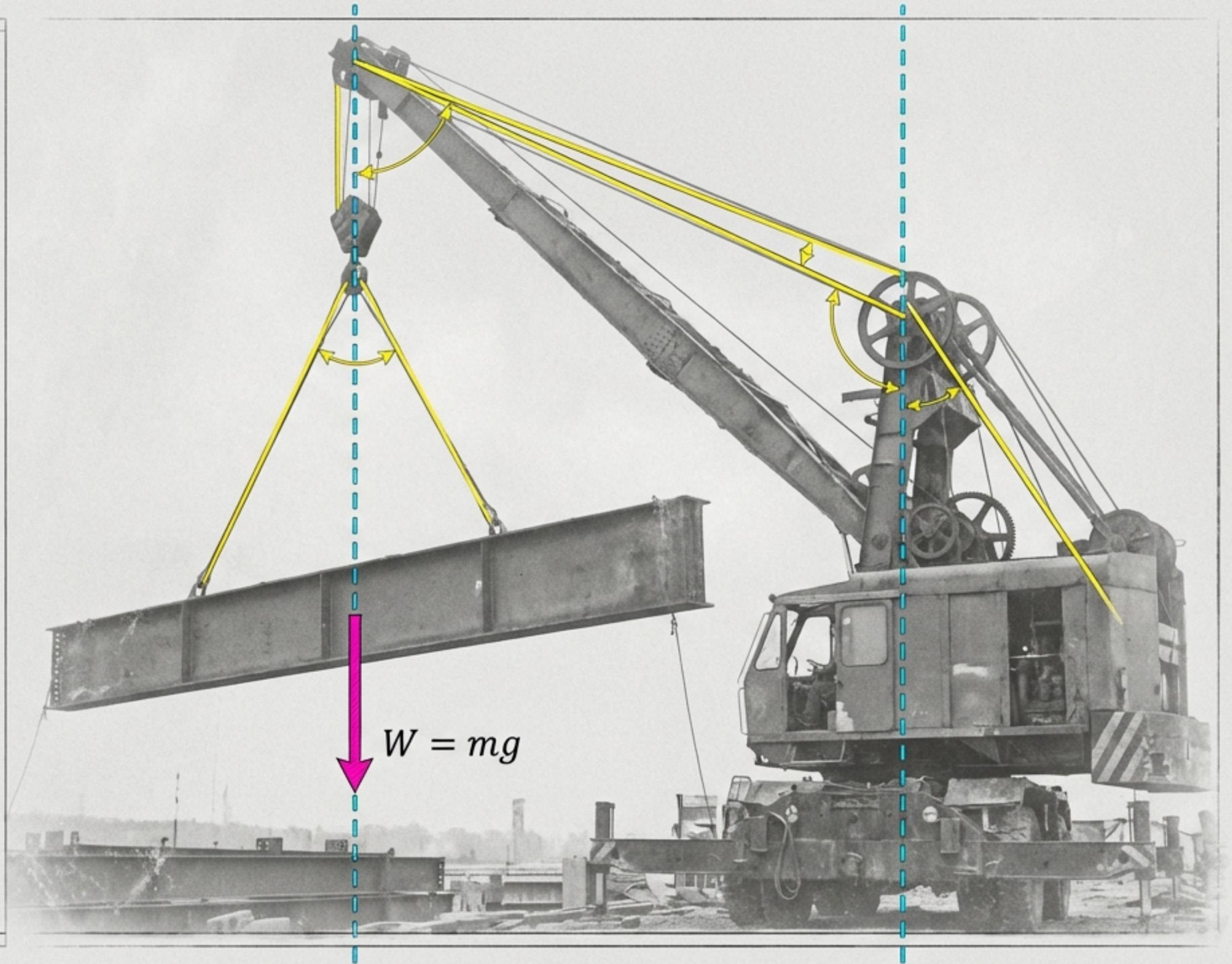


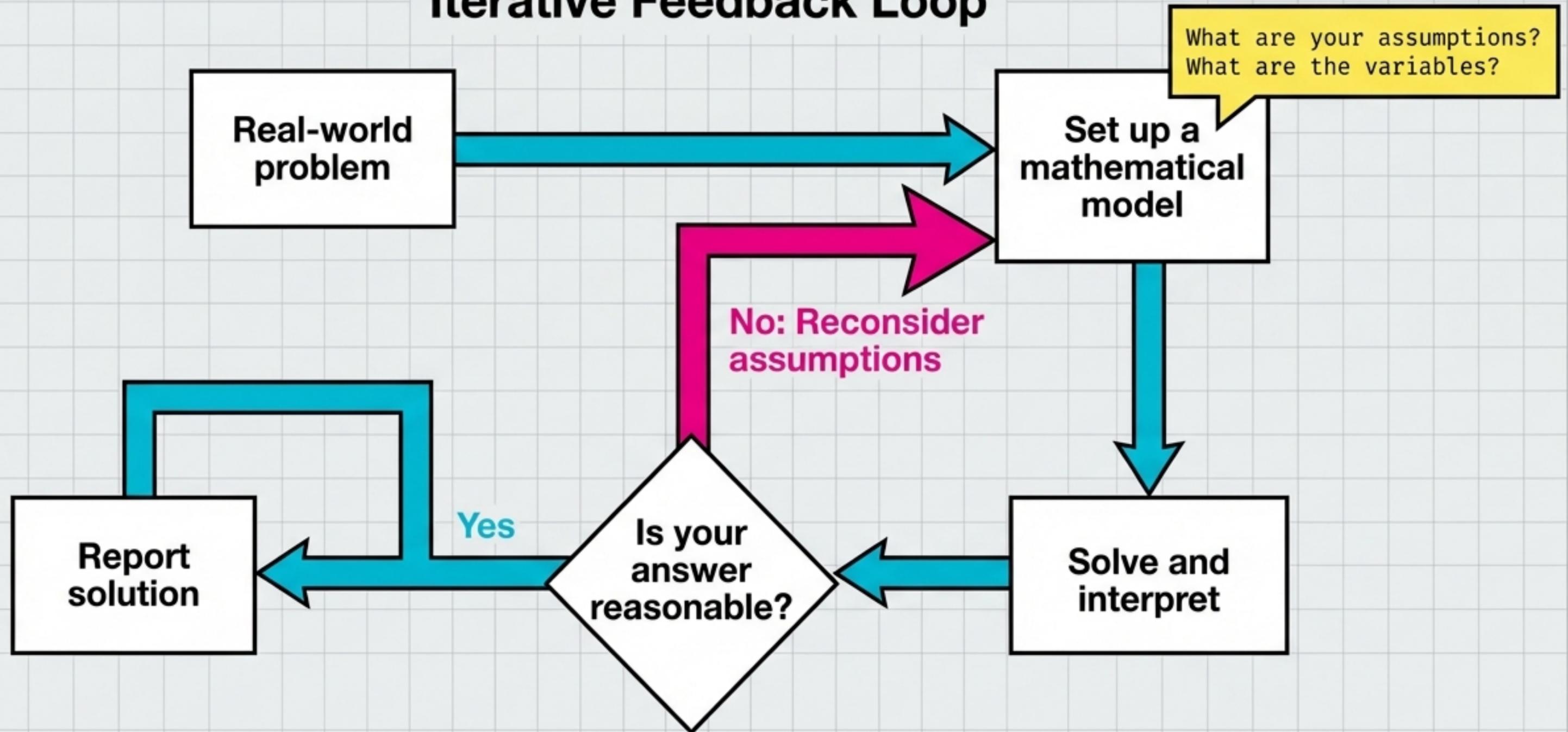
THE ART OF MATHEMATICAL MODELLING IN MECHANICS

Translating Reality into
Pure Mathematics

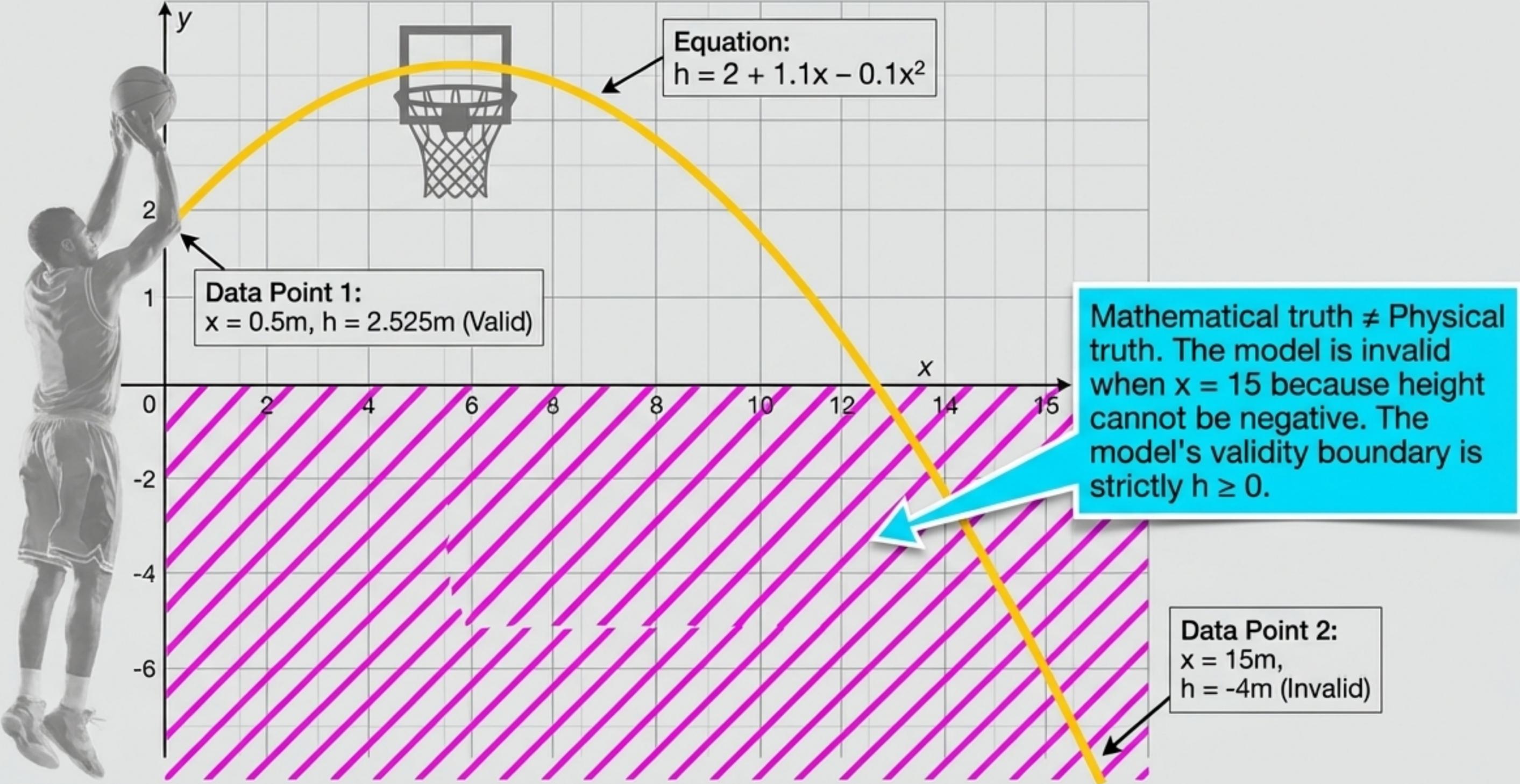


THE MATHEMATICAL MODELLING ENGINE

Iterative Feedback Loop

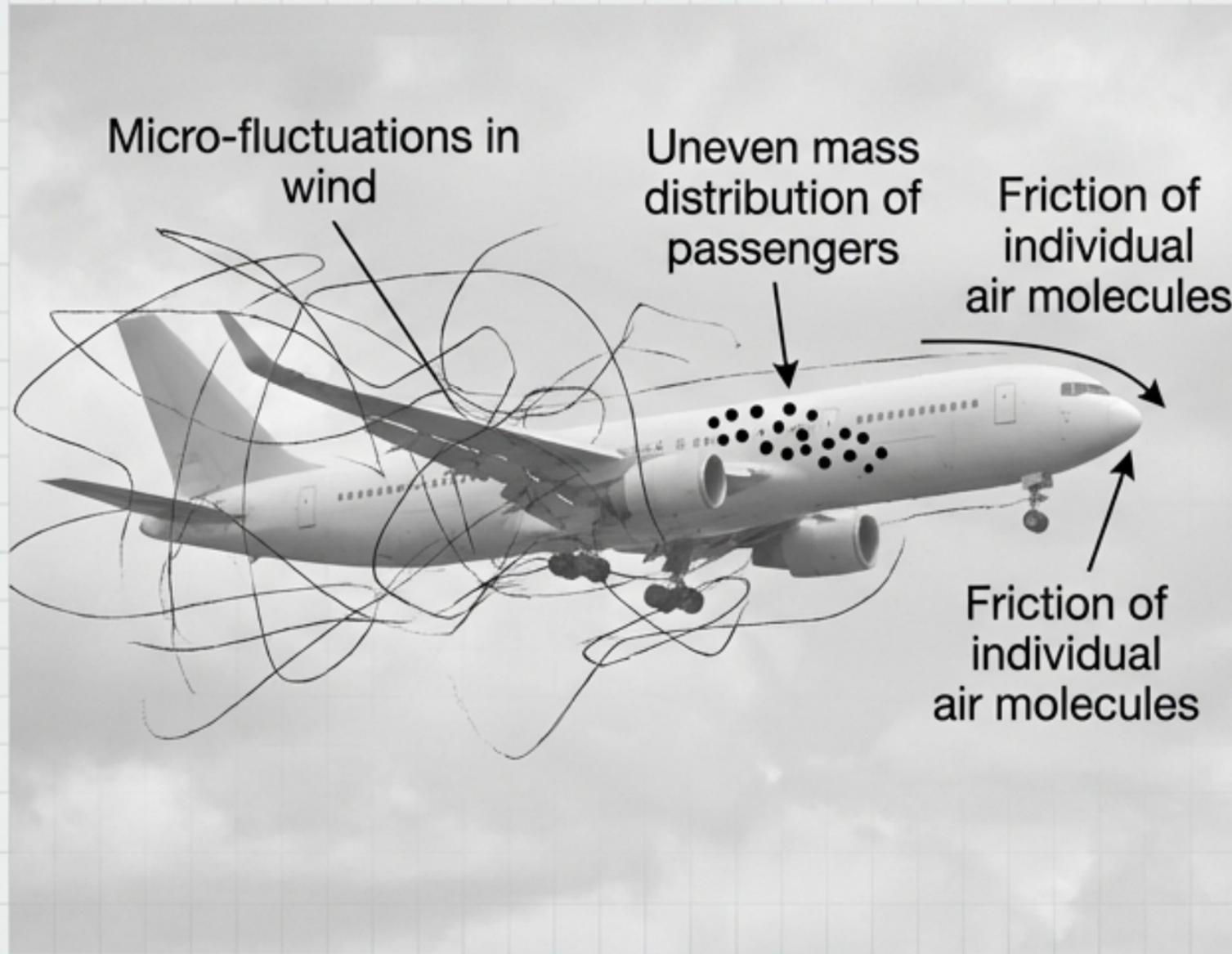


THE BOUNDARY OF VALIDITY

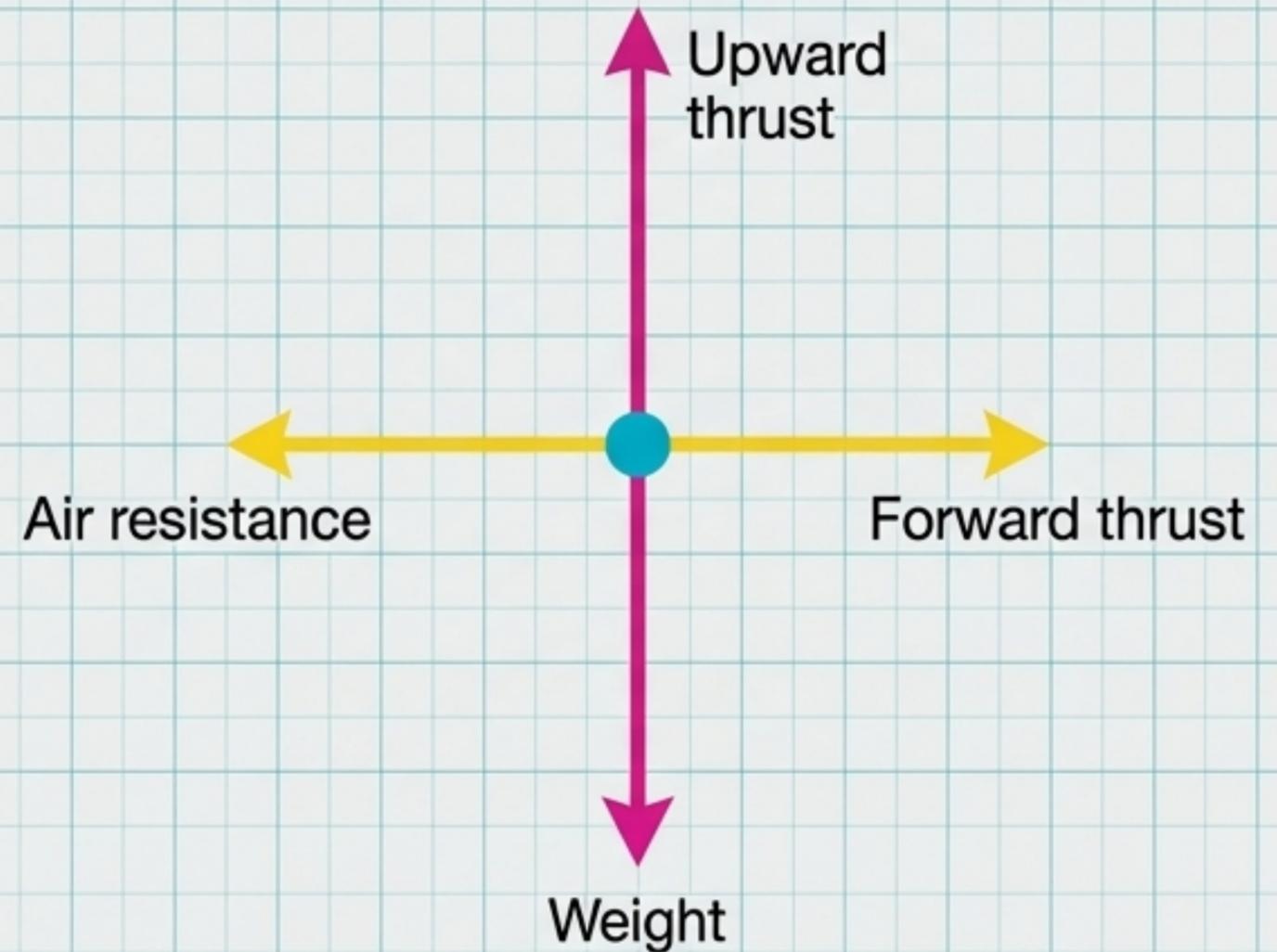


THE CORE CHALLENGE: REALITY VS. MODEL

REALITY



THE MODEL



To analyze reality, we must first aggressively simplify it. We do this through standard Modelling Assumptions—the syntax used to translate English into Mechanics.

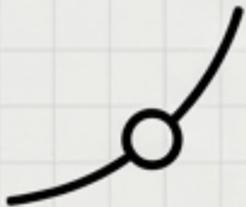
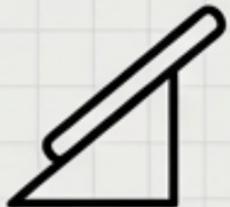
THE ASSUMPTIONS BLUEPRINT I: SIMPLIFYING OBJECTS

THE KEYWORD	MATHEMATICAL TRANSLATION	PROBLEM-SOLVING SUPERPOWER
Particle	Dimensions of the object are negligible.	Mass concentrated at a single point. Ignore rotational forces and air resistance.
Rod	All dimensions but one are negligible.	Mass concentrated along a line; rigid (does not bend or buckle).
Lamina	Object with area but negligible thickness.	Mass is distributed across a flat surface.
Uniform body	Mass is distributed evenly.	Mass is concentrated at a single point at the geometric centre of the body (centre of mass).
Light object	Mass of the object is small compared to other masses.	Treat object as having zero mass.

THE ASSUMPTIONS BLUEPRINT II: SIMPLIFYING ENVIRONMENTS

THE KEYWORD	MATHEMATICAL TRANSLATION	PROBLEM-SOLVING SUPERPOWER
Smooth surface	A surface on which it can be assumed there is no friction.	Assume there is absolutely zero friction between the surface and any object on it.
Rough surface	A surface on which there is friction.	Objects in contact with the surface experience a frictional force if moving or acted on by a force.
Air resistance	Resistance experienced as an object moves through the air.	Usually modelled as completely negligible unless specified in the question.
Gravity	Force of attraction between all objects.	Assume uniform acceleration ($g = 9.8 \text{ m s}^{-2}$) acting vertically downward on all mass everywhere.

THE ASSUMPTIONS BLUEPRINT III: SIMPLIFYING CONNECTIONS

	THE KEYWORD	MATHEMATICAL TRANSLATION	PROBLEM-SOLVING SUPERPOWER
	Inextensible string	A string that does not stretch under load.	Acceleration is exactly the same for all objects connected by a taut string.
	Smooth and light pulley	All pulleys you consider will be smooth and light.	Pulley has no mass. Tension is exactly the same on either side of the pulley.
	Bead	Particle threading on a wire or string.	Moves freely. For a smooth bead, tension is the same on either side.
	Peg	A support from which a body can be suspended or rested.	Dimensionless and fixed (can be rough or smooth as specified).

THE LANGUAGE OF MECHANICS: SI UNITS

BASE UNITS - THE FOUNDATION

Mass:
kilogram (kg)

**Length/
displacement:**
metre (m)

Time:
second (s)

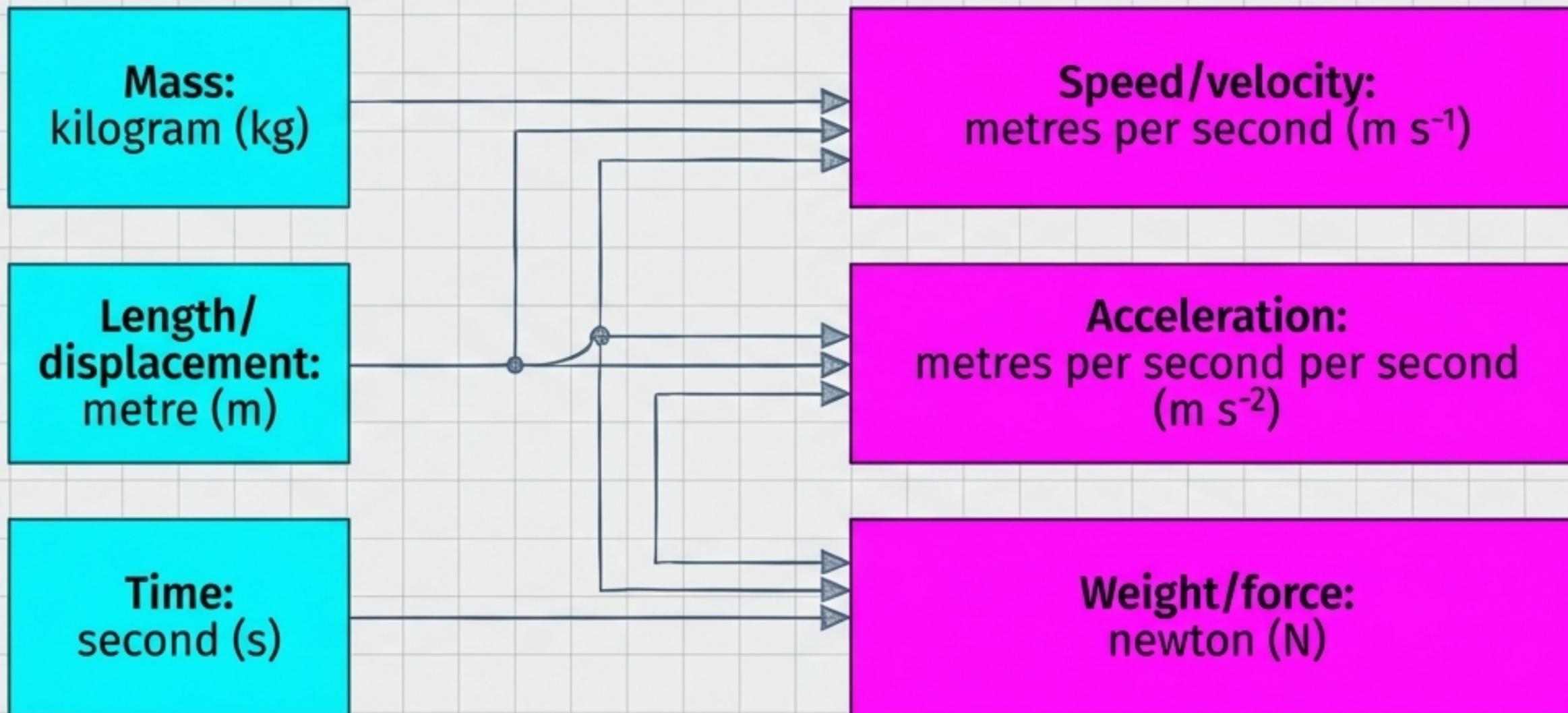
DERIVED UNITS - THE COMPOUNDS

Speed/velocity:
metres per second (m s^{-1})

Acceleration:
metres per second per second
(m s^{-2})

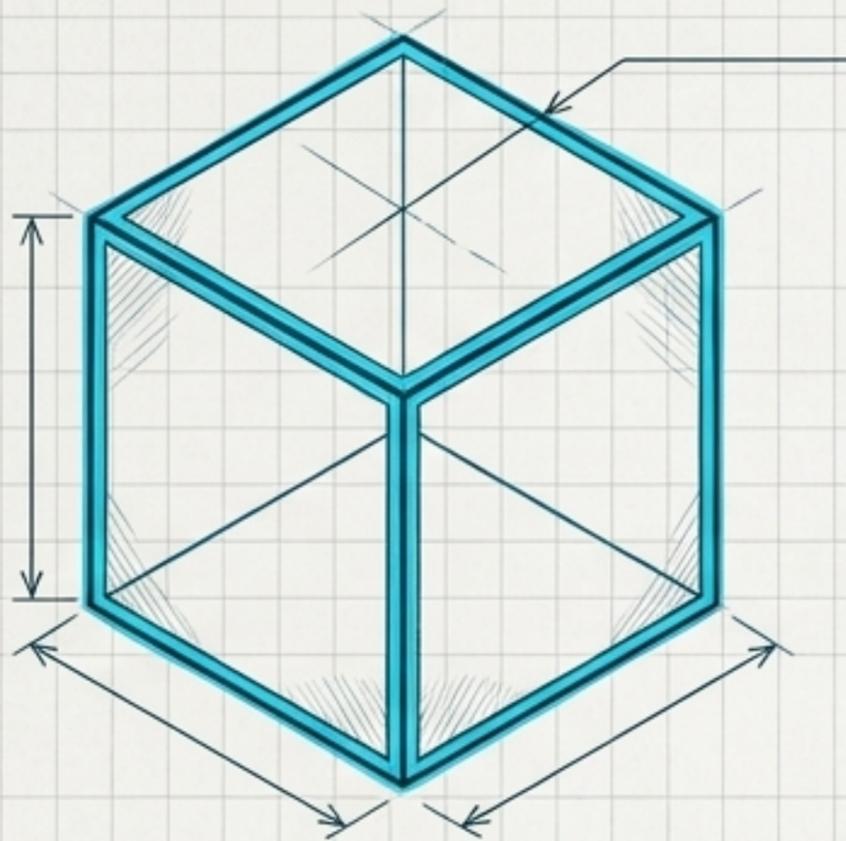
Weight/force:
newton (N)

$$\mathbf{N = kg\ m\ s^{-2}}$$



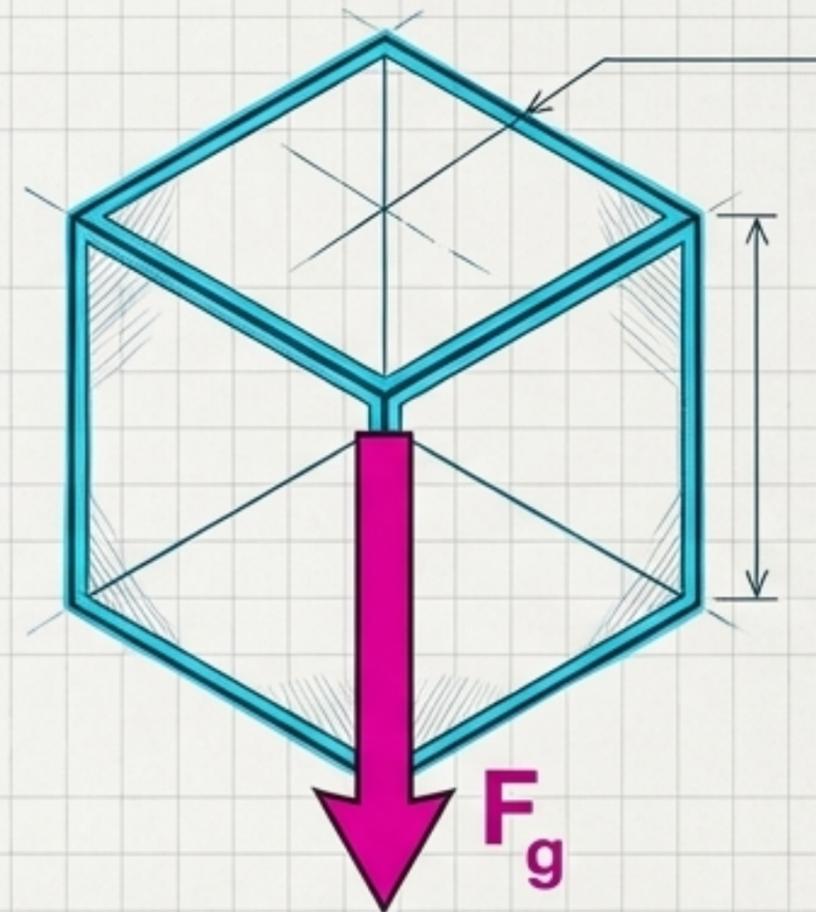
THE ULTIMATE TRAP: MASS VS. WEIGHT

MASS



- Intrinsic property of an object.
- Scalar quantity (has magnitude, no direction).
- Measured in kilograms (kg).
- Constant everywhere (same on Earth as on the Moon).

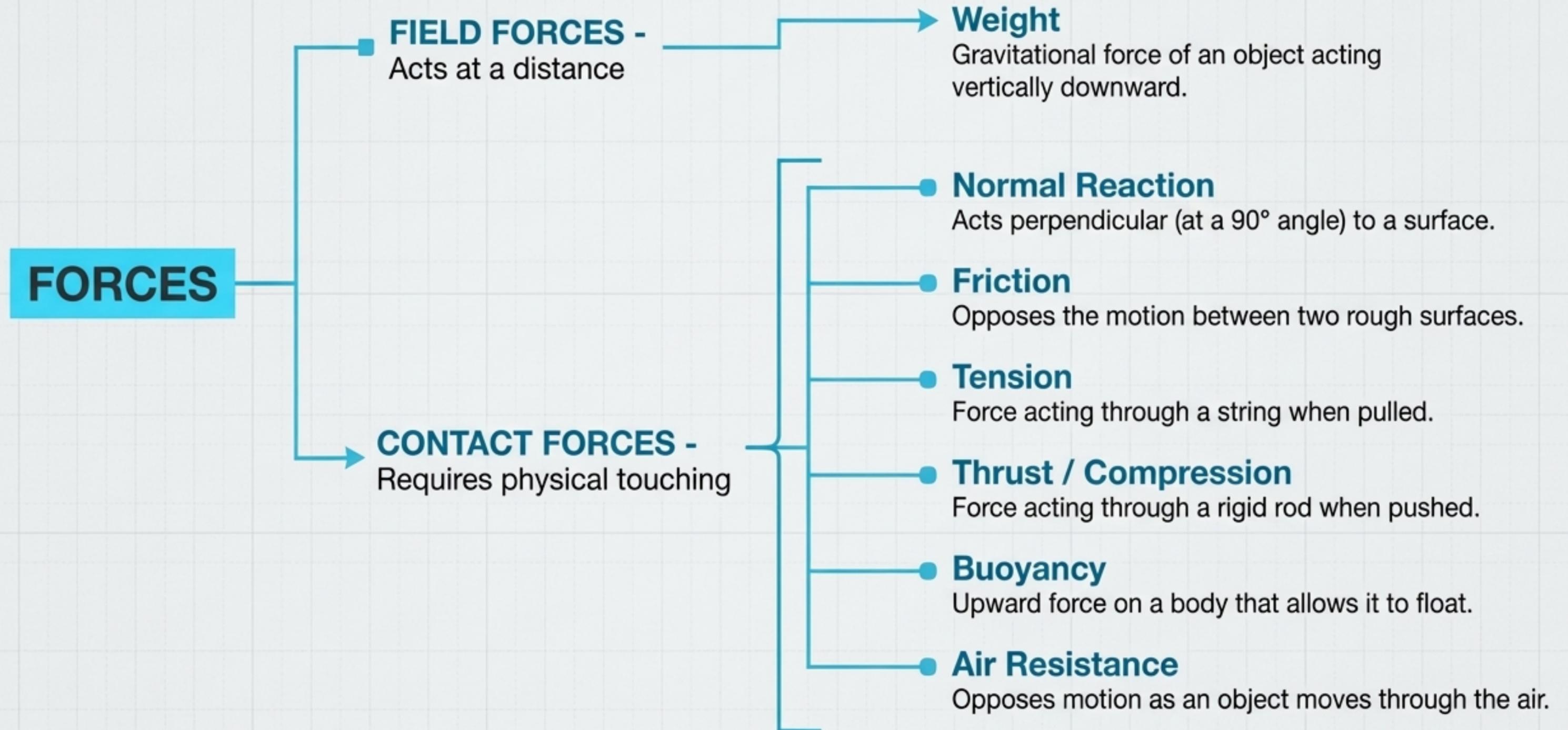
WEIGHT



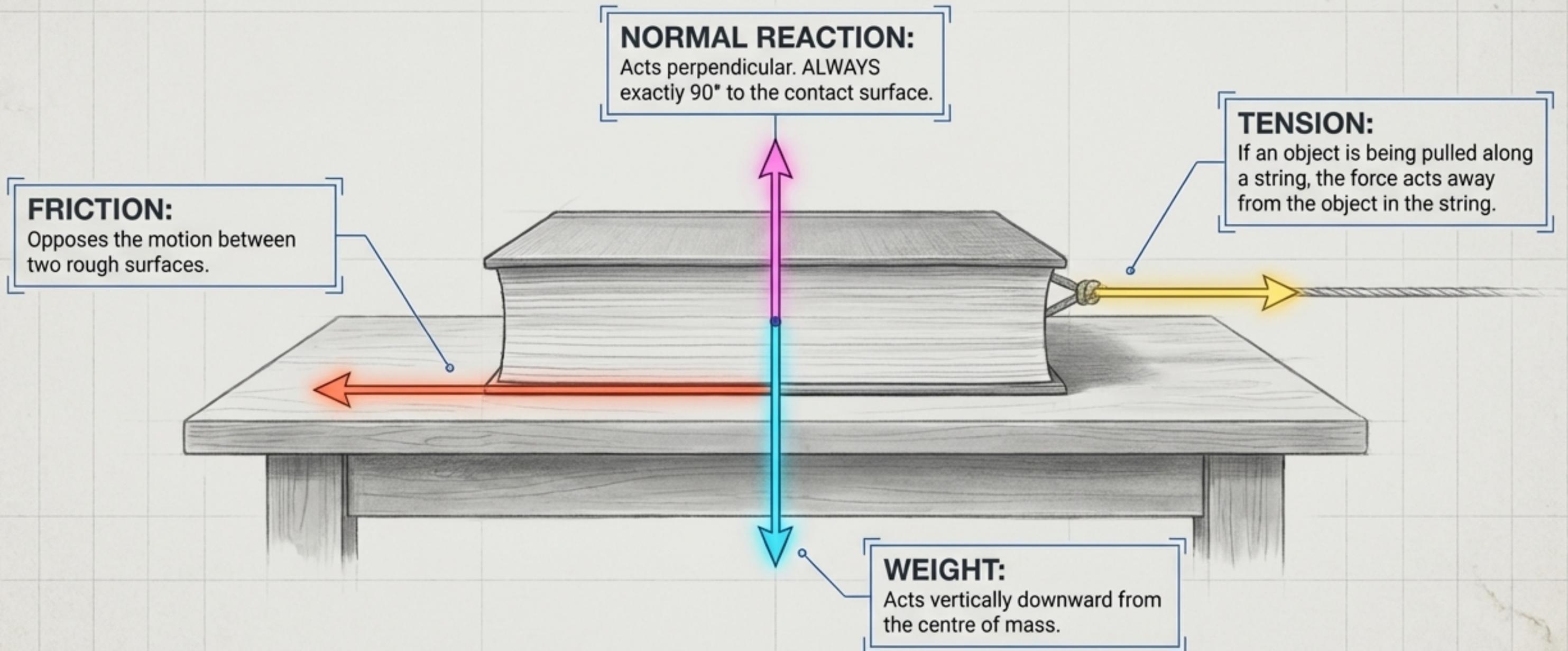
- Gravitational force acting upon an object.
- Vector quantity (has magnitude and downward direction).
- Measured in newtons (N).
- Changes depending on gravity ($W = mg$).

WATCH OUT: A common misunderstanding is that kilograms measure weight. They do not. Weight is a force.

THE GRAMMAR OF MOTION: TAXONOMY OF FORCES



VISUALIZING FORCES: ANATOMY OF A FORCE DIAGRAM



PROBLEM-SOLVING SYNTHESIS: TRANSLATING A SCENE

THE REALITY - ENGLISH

A mass is attached to a length of string which is fixed to the ceiling. The mass is drawn to one side with the **string stretched tightly** and allowed to swing.

THE MATHEMATICAL TRANSLATION - MATH

- **'A mass'** → Assume **Particle**. (Ignore the rotational effect of external forces and air resistance).
- **'string stretched tightly'** → Assume **Light and Inextensible**. (Ignore the mass of the string and any stretching effect).

